TEACHER'S GUIDE FOR

FMT INTERMEDIATE

Foundations of Mathematics For Tomorrow

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The Ryerson Mathematics Program

Teacher's Guide For

FMT Intermediate

Foundations of Mathematics
For Tomorrow

Table of Contents

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Teacher's Manual

Foundations of Mathematics for Tomorrow Intermediate

Dottori, Knill, and Stewart

General Notes

1) FMT has been written for the teacher, and for the student. Each section provides worked examples to illustrate the objectives. The teacher may use these as a guide in lesson preparation, and the student as a concise review of the topic.

   The written material is designed to present terminology and new ideas in a simple, straightforward way. If the objective is clearly stated by the teacher, many students will be able to proceed individually with a particular section, while the teacher assists those with difficulties. There is a need for students to develop the ability to use a textbook to learn, and for this reason, the assignment of selected sections should be an integral part of the teaching of the course.

2) The graded exercise in each section should allow the teacher to select sufficient problems for reinforcement of the basic objective. Many exercises also contain challenging problems for the more able student.

   Students should be encouraged to check all answers as they progress through the exercises.

3) Each chapter is preceeded by a review and preview section. Comments are made at the beginning of the teacher's manual on the use of these exercises. It may be advisable to be familiar with the content of these exercises when planning a course of study.

4) Interest pieces and puzzles have been inserted throughout the text. The student should be made aware of these early in the year, and should be challenged to attempt them when time is available. Invite

Review and Preview Summary.

Chapter (page)	Topics	Suggestions and notes.
1 (p.1)	Algebraic addition and subtraction, substitution, f(x) notation distributive property.	(i) Review or pretesting for Chapter 1 (ii) f(x) substitution used in section 1.7
2 (p.39)	Percentages, operations using BEDMAS.	(i) General computation review (ii) An opportinity to use the calculator.
3 (p.75)	Algebraic manipulation, continued fractions.	(i) Review of some ideas sections 1.12 and 2.3. (ii) continued fractions is an optional idea for the able student.
4 (p.98)	Slopes, coordinates of points on a line.	(i) Should be used as review prior to linear work of Chapter 4.
5 (p.120)	Algebraic addition and subtraction(vertical form), solving of linear equations in one variable.	(i) A good review prior to solution of linear systems of Chapter 5 where vertical addition and subtraction are used. (ii) Selected problems from Exercise 2 could be used as a review or test of equation solving techniques.

6 (p. 165)	Distance between two points (analytic geometry)	(i) Should be used as a review prior to the analytic geometry of Chapter 6.
7 (p.181)	Constructions in geometry, Angles (in triangles and involving parallel lines)	(i) Some constructions are used in Sections 8.6, 8.8 and the Review of 8 - these could be used as review at that time. (ii) The exercise on angles could be used orally to check student recall from past geometry work.
8 (p.223)	Networks	(i) An optional set of exercises of interest to the inquisitive, able student.
9 (p.261)	Matrices, matrix equations	(i) A review of concepts covered in Section 5.5 which are needed if Section 9.9 is to be studied.
10 (p.298)	Equations of lines, angles, congruence.	(i) The angle and congruence concepts will be necessary in Chapter 10. (ii) The equation review should be used prior to Section 10.6

11 (p.322)	Similar triangles, Pythagorean theorem, equations.	(i) All of these ideas will be used in Chapter 11.
12. (p.340)	Distance from the origin, primary trigonometric ratios.	(i) The distance calculation is necessary in Chapter 12. (ii) If the students have been previously introduced to angles in standard position, Exercise 2 is a good pretest.
13 (p.364)	Computation	(i) This exercise is designed for calculator practice. (ii) Problem 8 poses the additional problem of obtaining a set of the discs and attempting to complete the moves in the minimum calculated number of steps. Don't try too many discs!

Chapter 1 Algebra

1.1 Product of Polynomials 1 period

A) Objective: To multiply polynomials.

B) Notes: (a) Most of the terminology and methods of this section should be familiar to the student. A pretest may show that students are competent in the special binomial cases.

(b) The distributive property is used to illustrate the concept of "multiplying each term in one polynomial by each term in the other."

(c) Competence in multiplying binomials aids in factoring trinomials in section 1.3.

C) Selected problems should provide the necessary practice. The more able student may wish to investigate the pattern illustrated below:

		coefficients
$(x+1)^0 =$	1	1
$(x+1)^1 =$	$1x+1$	1 1
$(x+1)^2 =$	$1x^2+2x+1$	1 2 1
$(x+1)^3 =$	$1x^3+3x^2+3x+1$	1 3 3 1
$(x+1)^4 =$	by studying the pattern at right, predict the products.	
$(x+1)^5 =$		

1.2 Common Factor 1 period

A) Objective: To factor polynomials containing a common factor in each term.

B) Notes:

(a) Review the distributive property. Stress that factoring is the opposite to expanding.

$$3(x+2) = 3x+6$$

(b) Emphasize that factoring is complete when no more variable or integral factors can be removed.

$$4x^2 + 8x$$
$$= 4(x^2 + 2x)$$

variable factor remaining

(c) All factoring problems may be checked quickly by expanding!

(d) Note example 3 as an introduction to factoring by grouping.

$$10x^2 - 5xy - 6x + 3y$$
$$= (10x^2 - 5xy) - (6x - 3y)$$

Note the sign change necessitated by inserting brackets.

The order of terms may be varied in a grouping problem.

$$10x^2 - 6x - 5xy + 3y$$
$$= (10x^2 - 6x) - (5xy - 3y)$$
$$= 2x(5x - 3) - y(5x - 3)$$
$$(5x - 3)(2x - y)$$

C) Use selected problems from 1 to 4. The more able student should attempt problem 6.

1.3 Factoring $ax^2 + bx + c$ 2 periods

A) Objective: To factor trinomials.

B) Notes: (a) The basis for the grouping method is illustrated as follows:

$$(px+r)(qx+s)$$
$$= pqx^2+psx+qrx+rs$$
$$= pqx^2 + (ps+qr)x + rs$$

(1) the product of these factors is pqrs

(2) the product of these is also pqrs

In a given trinomial, $6x^2 + 19x + 8$, the product (1) is $6 \times 8 = 48$.

The problem is to find two factors of 48, which will add to +19.

$$\begin{aligned} 48 &= 6 \times 8 \\ &= 2 \times 24 \\ &= 4 \times 12 \\ &= 3 \times 16 \quad * \; 3 + 16 = 19 \\ &= 48 \times 1 \end{aligned}$$

$$\begin{aligned} & 6x^2 + 19 + 8 \\ &= 6x^2 + 3x + 16x + 8 \\ &= 3x(2x + 1) + 8(2x + 1) \\ &= (2x + 1)(3x + 8) \end{aligned}$$

use 3 and 16 to decompose the middle term. The order does not matter.

(b) Analysis and trial is also a useful method of factoring. This depends on the student's competence in multiplication, and on allowing sufficient time for practice.

e.g. $6x^2 + 13x - 5$ has factors of the form $(3x \ldots)(2x\ldots)$ or $(6x\ldots)(x\ldots)$

The possibilities are

*$(3x \quad 1)(2x \quad 5)$	or	$(6x \quad 1)(x \quad 5)$
$(3x \quad 5)(2x \quad 1)$		$(6x \quad 5)(x \quad 1)$

* will have inner terms of $15x$ and $2x$ when expanded, and will leave a middle term of $13x$ if $15x$ is positive, and $2x$ is negative.

$$6x^2 + 13x - 5 \text{ must have factors } (3x - 1)(2x + 5).$$

(c) $\boxed{\text{In all factoring problems, emphasize that common factors should be removed first.}}$

C Sufficient problems should be selected to allow the student to develop <u>competence</u> in factoring trinomials. Encourage the use of both methods!

1.4 <u>Factoring Special Quadratics</u> 1 period

A. Objective: (i) To factor a perfect square, and the difference of squares.

(ii) To factor by grouping.

B. Notes:

(a) Review briefly the <u>recognition</u> of perfect squares such as $4x^2 + 20x + 25$, and difference of squares such as $9x^2 - 16$.

(b) Use this recognition to help the student to see:

(i) the perfect square in $x^2 + 6x + 9 - y^2$

and (ii) the difference of squares form after grouping

e.g. $(x^2 + 6x + 9) - y^2$

$= (x + 3)^2 - y^2 \qquad \square^2 - \square^2$

$= (x + 3 - y)(x + 3 + y)$

(c) In grouping, note that sign changes are necessary in subtraction.

e.g. $a^2 - x^2 + 4x - 4$

$= a^2 - (x^2 - 4x + 4)$ note!

$= a^2 - (x - 2)^2$

$= [a + (x - 2)][a - (x - 2)]$

$= (a + x - 2)(a - x + 2)$ note!

C. Problems 1 and 2 may be done orally. Do a reasonable number of parts from questions 4 and 5 to expose the student to factoring by "grouping".

1.5 Dividing a Polynomial by a Monomial 1 period

A. Objective: To divide a polynomial by a monomial.

B. Notes:

(a) Illustrate the concept of factoring in division.

e.g. $\frac{5x + 10}{5}$

$= \frac{5(x + 2)}{5}$

$= x + 2$

In practice, it is still advantageous to simplify as in example 2 and 3 where each term is divided by the monomial.

C. Most of the problems may be done orally in this exercise.

1.6 Dividing a Polynomial by a Polynomial 1 period

A. Objective: To divide a polynomial by a polynomial.

B. Notes: (a) Remind students that division by zero is undefined. This leads to restrictions on the variables:

i.e. Dividing by $x - 5$ implies $x \neq 5$

(b) In Example 3, note the use of the term "$0x^3y$" to ensure that like algebraic terms are in the same column for subtraction.

C. Emphasize selections from problems 1 to 4.

1.7 <u>The Remainder Theorem</u> 1 period

A. Objective: To prove and apply the Remainder Theorem.

B. Notes: (a) The emphasis should be on the use of the theorem for divisors of the form $x - b$, rather than $ax - b$.

(b) It may be necessary to review the substitution process.

i.e. $P(x) = x^2 + 5x + 2$ $P(3) = (3)^2 + 5(3) + 2$

Use problems 1, 2, and 3 for this purpose.

(c) Example 2 and problem 5 may be considered optional. However, students involved in mathematics contests should attempt these problems.

C. Emphasize problems 1 to 4.

1.8 <u>The Factor Theorem</u> 2 periods

A. Objective: To factor polynomials using the Factor Theorem.

B. Notes: (a) Emphasize the use as a factoring method. The application to equation solving is illustrated in Section 2.6, Example 4.

(b) Stress that factoring must be complete!

i.e. $(x+2)(x^2+7x+12)$ ← $x^2+7x+12$ can be factored

$= (x+2)(x+4)(x+3)$

(c) The substitution in $P(x)$ should be done in an organized manner. i.e. $P(1)$, $P(-1)$, $P(2)$, $P(-2)$, ...
and should be done <u>carefully</u>.

If $P(x) = x^3 + 2x^2 + 2x + 1$, then only $P(-1) = 0$.
If $P(-1)$ is evaluated incorrectly, the student will not be able to factor $P(x)$ over the integers.

C. Emphasize problems 1 to 7. (7i,j require more than one application of the Factor Theorem.) Problems 8 to 11 deal with the factoring of the sum, and difference, of cubes - an exercise for the more able student. Problems 12 to 14 apply the ideas from Example 2.

1.9 <u>Simplifying Rational Expressions</u> 1 - 2 periods

A. Objective: To simplify rational expressions.

B. Notes: (a) Note the need for restrictions.

in $\frac{(x-3)(x+3)}{(x-3)(x+2)}$ ← from this line, there are <u>two</u> divisors

$= \frac{x+3}{x+2}$ $\therefore$ $x \neq 3$ or -2

(b) Emphasize the idea from Example 3:

i.e. $\frac{x-3}{3-x} = \boxed{\frac{x-3}{(-1)(x-3)}}$ ← this step may be omitted when the idea is mastered.

$= -1$

(c) Note the common errors outlined on p. 27.

(d) Above all, for this section and the next three,

<u>emphasize the necessity of factoring first</u>. This will help avoid some of the common errors!

C. Problems 1 to 3 are suitable for oral discussion. Assign problems 4 and 5.

Problem 6 poses more complicated ideas and prior examples may be necessary before assigning.

1.10 <u>Multiplication and Division of Rational Expressions</u> 2 periods

A. Objective: To multiply and divide rational polynomial expressions.

B. Notes: (a) Review simpler multiplication and division using problem 1 of the exercise.

(b) Stress the necessity of factoring!

(c) To avoid incorrect "cancelling" in division, invert the divisor before factoring as in Example 2.

C. Assign selections from problems 1 to 5. For the factoring in problem 5, suggest that any work required be done at the side rather than in the middle of the problem.

e.g. $\dfrac{x^2 - 4}{x + 2} \times \dfrac{3x^2 + 5x - 2}{x^2 - 2x}$

$= \dfrac{(x - 2)(x + 2)}{x + 2} \times \dfrac{(3x - 1)(x + 2)}{x(x - 2)}$

$= \dfrac{(x+2)\ (3x-1)}{x}, \quad x \neq -2, 2$

<u>Factoring</u>

$3x^2 + 5x - 2 \qquad (3)(-2) = -6$

$= (3x^2 - 1x) + (6x - 2) \qquad -1, +6$

$= x(3x - 1) + 2(3x - 1)$

$= \underbrace{(3x - 1)(x + 2)}$

Insert these factors into solution at left.

1.11 <u>Least Common Multiple</u> 1 period

A. Objective: To determine the LCM for polynomial expressions.

B. Notes: (a) This is a necessary concept for determing the lowest common denominator in Section 1.12.

C. Emphasize problems 1 to 3.

1.12 <u>Addition and Subtraction of Rational Expressions</u> 2-3 periods

A. Objective: To add and subtract rational expressions.

B. Notes: (a) Stress the use of the LCM to obtain the least common denominator.

(b) Note the use of brackets in the examples.

e.g. Example 2

$$\frac{(2w-3)}{4} + \frac{(3w-1)}{5} - \frac{(w-5)}{2}$$

bracket the numerators first.

$$= \frac{5(2w-3)+4(3w-1)-10(w-5)}{20}$$

$$= \frac{10w-15+12w-4-10w+50}{20}$$

Remind students to
(i) use the distributive property.
(ii) obtain correct signs in subtraction.

C. Assign problems from 1 to 7. Problem 8 combines operations to provide more challenging questions.

A thought for problems such as 8(e)

$$\frac{1-\frac{2}{x}+\frac{3}{x^2}}{2-\frac{1}{x}-\frac{1}{x^2}} = \frac{\left(1-\frac{2}{x}+\frac{3}{x^3}\right) x^2}{\left(2-\frac{1}{x}-\frac{1}{x^2}\right) x^2}$$

multiply numerator and denominator by the LCD.

$$= \frac{x^2-2x+3}{2x^2-x-1}$$

now factor and simplify.

Chapter 2   <u>The Real Numbers</u>   1 period

2.1 <u>The Number System</u>

A. Objective: To define and graph sets of real numbers.

B. Notes: (a) Review set development briefly i.e. N, W, I and Q.

(b) Use the method of Example 2 to show that a periodic decimal can be written in the form $\frac{a}{b}$, $b \neq 0$ which is the definition for a rational number.

(c) Stress that $-2 \leq x \leq 3$ is equivalent to $x \geq -2$ <u>and</u> $x \leq 3$.

(d) Review: $A \cup B = \{x \mid x \in A \text{ or } x \in B \text{ or } x \in A \text{ and } B\}$

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

(e) Note the method in Example 4 where each set is graphed, and from these graphs the union and intersections can easily be seen. Overhead examples assist greatly in presenting examples such as these.

C. Use selected problems from 1 to 6. In problem 2, the student may be encouraged to note a pattern in the results.

e.g. $0.\dot{3}\dot{6} = \frac{36}{99}$ $0.\dot{5} = \frac{5}{9}$ Add a few of the type $0.2\dot{3}$, $0.6\dot{5}$, $0.2\dot{3}6\dot{5}$ to see if they can discover this pattern.

number of 9's equal to length of period

Problems 7 to 10 may be assigned to the more able student. The results for problem 8 are interesting.

2.2 <u>Properties of Real Numbers</u> 1 period

A. Objective: To familiarize the student with the properties of real numbers.

B. Notes: (a) A prepared overhead showing the form of the properties should be displayed while numerical examples are used to illustrate the meaning.

(b) Encourage the student to note this section so that it can be referred to as the properties occur in examples throughout the text.

(c) Interested students should prepare a set of posters to permanently display these properties in the classroom.

C. Selected problems should be orally done with the class (with the properties displayed on an overhead.)

2.3 <u>Linear Equations</u> 1 period

A. Objective: To solve linear equations.

B. Notes: (a) This should be review for all students.

(b) Note particularly the careful steps to be taken in Example 1(c)

(i) $6 \times \frac{1}{3}(2x - 5) - 6 \times \frac{1}{2}(x + 3) = 2 \times 6$

Watch that student
does <u>not</u> introduce a distributive property for multiplication. i.e. $6 \times \frac{1}{3} \times 6(2x - 5)$

(ii) $2(2x - 5) - 3(x + 3) = 12$

$4x - 10 - 3x - 9 = 12$

watch this sign!

C. Use a suitable selection of problems from 1 to 4. Problem 5 is a useful challenge for students at this level.

2.4 Linear Inequalities 1 period

A. Objective: To solve linear inequalities.

B. Notes: (a) The concept of reversing the inequality symbol when multiplying or dividing both sides by a negative should be stressed.

e.g. $-3x < -15$

$\frac{-3x}{-3} > \frac{-15}{-3}$ — the symbol is reversed in this step - not after the division is complete.

$x > 5$

C. Use selected problems from 1 to 5 to practice these ideas. Problems 6 and 7 will challenge the more able student.

2.5 Absolute Value 1-2 periods

A. Objective: To define the absolute value of n, and to apply the definition to solving equations and inequalities.

B. Notes: (a) The steps in solving examples must be carefully developed particularly in the solution of inequalities.

e.g. Example 3 Solve and graph $|x - 4| \leq 2$, $x \in R$

Solution: Either

(i) $(x - 4) < 0$ implies $x < 4$ OR (ii) $(x - 4) \geq 0$ implies $x \geq 4$

definition $\therefore |x - 4| = -(x - 4)$ $\therefore |x - 4| = x - 4$

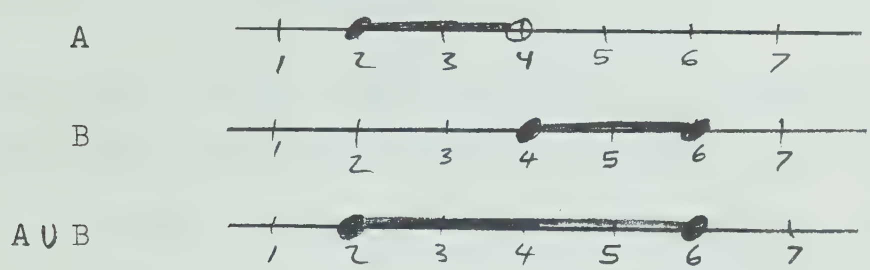
but since $|x - 4| \leq 2$, but since $|x - 4| \leq 2$

substitution $\therefore\ -(x-4) \leq 2$ $\qquad$ $\therefore\ x - 4 \leq 2$

$x \geq 2$ $\qquad$ $x \leq 6$

$\therefore\ (x \geq 2 \text{ and } x < 4)$ OR $(x \leq 6 \text{ and } x \leq 4)$

The solution set is the union of A and B. $\qquad A \cup B$



$\therefore$ the solution set is $\{x \mid 2 \leq x \leq 6\}$

(b) Some concepts of this section may not be suitable for all students at this level.

C. Problems 1 and 2 are suitable for oral discussion. Select sufficient parts from problems 3 and 4 to practice the concepts. The definition following problem 4, and problems 5 and 6 should be assigned to the more able student.

2.6 <u>Quadratic Equations Solved by Factoring</u> 2-3 periods

A. Objective: To solve equations of degree 2 or 3 by factoring.

B. Notes (a) Introduce terminology

Equation	degree	name	number of roots
$x^1 + 5 = 7$	1	linear	
$x^2 + 3x + 2 = 0$	2	quadratic	
$x^3 - 9x = 0$	3	cubic	

this column can be completed as examples are studied.

(b) Emphasize that one side must be zero for the factoring concept to apply. This is illustrated in Example 2 and 3. Stress the simplification of equations before attempting to factor.

(c) The solving of quadratic equations should be the main emphasis at this level.

C. Stress problems 1 to 4 as a first assignment. When complete, and if time permits, Example 4 and problem 5 may be covered.

Problems 6, 7 (an interesting challenge), 8, and 9 may be assigned for the more able student.

2.7 Quadratic Equations Solved by Formula 2 periods

A. Objective: To solve quadratic equations by formula.

B. Notes: (a) The method used to develop the formula is numerically illustrated in Examples 1 and 2. Since a perfect square is required on the left side, some practice should be provided in "completing the square".

e.g. State the value to be added to complete the square.

$x^2 + 6x +$ ___ develop the idea that

$x^2 + 18x +$ ___ this value is equal to

$(\frac{1}{2}$ coefficient of $x)^2$

(b) An alternate development of the quadratic formula:

$$ax^2 + bx + c = 0, \quad a \neq 0$$

$$ax^2 + bx = -c$$

$$\times 4a \rightarrow \quad 4a^2x^2 + 4abx = -4ac$$

$$+ b^2 \rightarrow \quad 4a^2x^2 + 4abx + b^2 = b^2 - 4ac$$

$$(2ax + b)^2 = b^2 - 4ac$$

$$2ax + b = \pm\sqrt{b^2 - 4ac}$$

$$2ax = -b \pm \sqrt{b^2 - 4ac}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(c) Note that $a \neq 0$ since $ax^2 + bx + c = 0$ would become $bx + c = 0$ which is a linear equation.

(d) In using the formula, the right side of the equation must be zero.

(e) Illustrate that solving by factoring should be attempted first since the solution is usually simpler.

e.g. $2x^2 + 7x + 3 = 0$

$(2x + 1)(x + 3) = 0$

$\therefore \quad x = -\frac{1}{2}$ or $x = -3$

OR

$2x^2 + 7x + 3 = 0$

$a = 2, \ b = 7, \ c = 3$

$$x = \frac{-7 \pm \sqrt{49 - 24}}{4}$$

$$= \frac{-7 \pm \sqrt{25}}{4}$$

$$= \frac{-7 \pm 5}{4}$$

$x = \frac{-12}{4}$ or $x = \frac{-2}{4}$

$x = -3$ or $x = -\frac{1}{2}$

C. Select sufficient problems from 1 to 15 to allow student to develop competence in using the formula.

TEST THE SOLVING OF QUADRATICS AT THIS POINT.

2.8 <u>Quadratic Inequalities</u> 1 period

A. Objective: To solve and graph quadratic inequalities.

B. Notes: (a) Note the use of factoring to solve inequalities - one side must equal zero.

(b) Careful study of Examples 1 and 2 should illustrate the necessary steps.

C. Assign carefully selected problems from 1, 2, and 3. Problem 4 is for the more able student since more cases must be examined.

e.g. 4(a) (i) $x < 0$ and $x - 1 > 0$ and $x + 3 > 0$

OR

(ii) $x < 0$ and $x - 1 < 0$ and $x + 3 < 0$

OR

(iii) $x > 0$ and $x - 1 < 0$ and $x + 3 > 0$

OR

(iv) $x > 0$ and $x - 1 > 0$ and $x + 3 < 0$

2.9 <u>Radical Expressions</u> 1 period

A. Objective: To simplify radical expressions.

B. Notes: (a) Emphasize radicals with index 2 understood. e.g. $\sqrt{3}$, and the fact that in this case the symbol, $\sqrt{\ }$, represents the principal (positive) square root of the radicand.

This leads to two basic ideas

(i) since the index is even, the radicand must be positive

e.g. $\sqrt{x - 2}$ implies $x \geq 2$

$\sqrt{b}$ implies $b \geq 0$

(ii) $\sqrt{x^2y} = \sqrt{x^2} \times \sqrt{y}$ $\quad y \geq 0$

$= |x|\sqrt{y}$

x may be negative, which is acceptable in the original since $x^2 \geq 0$ always.

the absolute value bars ensure a positive value for $\sqrt{x^2y}$

(b) Most of the ideas in the examples should be a review for students.

C. Problems 1 to 5 are suitable for oral discussion. Assign a suitable selection from problems 6 to 10. (10g,h,i are useful for Section 2.10). Problem 15 may be assigned at this time, or in conjunction with Section 2.11.

2.10 Rationalizing the Denominator 1 period

A. Objective: To rationalize the denominator in radical expressions.

B. Notes: (a) The use of calculators has reduced the need for this operation as a method of simplifying computation.

C. A suitable assignment might contain 2b,c; 3b,e,f,h; and 4a,c,f,i.

2.11 Radical Equations 1-2 periods

A. Objective: To solve and verify radical equations.

B. Notes: (a) In determining the domain of the variable, two ideas must be considered:

(i) in $\sqrt{(\text{[redacted]})}$, the expression ([redacted]) ≥ 0

(ii) division by zero is undefined. (Example 1(d))

(b) Example 2 provides an initial illustration, but before considering Example 3, provide an additional example to illustrate an extraneous root.

e.g. $\sqrt{x+12}+6 = 2 \qquad\qquad x+12 \geq 0 \quad \therefore \quad x \geq -12$

$$\sqrt{x+12} = -4$$

$$x+12 = 16$$

$$x = 4 \qquad \text{note: } 4 \geq -12$$

Verify $\underline{LS}$: $\sqrt{4+12}+6 \qquad\qquad \underline{RS}$:

$= \sqrt{16}+6 \qquad\qquad 2$

$= 10$

$\therefore$ 2 is an extraneous root.

In this example, the line $\sqrt{x+12} = -4$ should be noted since the symbol, $\sqrt{}$, represents a <u>positive</u> root. However, this cannot be seen in a line such as

$\sqrt{2x-5} = x-4$ since we do not know if $x-4$ is positive or negative.

(c) In Example 3, stress the careful squaring of the expression $(3-\sqrt{x+2})$ as a <u>binomial</u>.

e.g. $(3-\sqrt{x+2})^2 = (3-\sqrt{x+2})(3-\sqrt{x+2})$

$$= 9 - 6\sqrt{x+2} + (x+2)$$

See problem 15, Exercise 2-9 for practice in this step!

C. Problems 1 and 2 provide oral practice. A suitable selection should be made from problems 3,4, and 5.

2.12 <u>Applications of Real Numbers</u> 2-3 periods

A. Objective: To solve a variety of problems using the concepts of this chapter.

B. Notes: (a) The formulas $D = RT$, $R = \frac{D}{T}$ and $T = \frac{D}{R}$ should be reviewed prior to considering Example 2.

(b) Stress the use of statements to introduce the quantities and units represented by variables.

(c) Example 3 and the related problems 19 and 20 may be optional depending on the emphasis given to Section 2.8.

C. The exercise may be split into three sections:

(i) Problems 1,2,4,5,6,7 - a variety of real number problems.

(ii) Problems 8,9,10,11,12,13 - word problems involving the ideas of Example 1 plus area concepts.

(iii) Problems 3, 14,15,16,17,18 - problems involving distance, rate and time.

Chapter 3 FUNCTIONS

3.1 Function Notation 2 periods

A. Objective: To define a function, and use various function notations.

B. Notes: (a) Note from the definition, that if A is the domain of "f", then each element $x \in A$ is associated with a value f(x) in the range of "f". See Example 2(c).

C. The examples and exercise problems may be taught in two sections.

(i) Introduction and Examples 1 and 2 followed by selected parts of problems 1,2,3 oral
5,6,7,8,9 (problem 10 may be included)

(ii) Examples 3,4,5,6 followed by problems 4 oral, 11,12,13,14,15

Problems 17,18,19,20, if assigned should be preceded by examples.

e.g. Given $f(x) = x^2 + 2$ and $g(x) = x - 1$, determine

(a) $g[f(2)]$ (b) $f[g(x)]$

Solutions

(a) $f(2) = 2^2 + 2$
$= 6$
$g(6) = 6 - 1$
$= 5$

(b) $f[g(x)] = f(x - 1)$ substitute for x in f(x)
$= (x - 1)^2 + 2$
$= x^2 - 2x + 3$

3.2 Graphs of Functions 1 period

A. Objective: To graph functions.

B. Notes: (a) Stress that the domain of the function must be noted when graphing. This is illustrated in Examples 1,2 and 4.

(b) Note that the statement $f(3) = -2$ implies an ordered pair $(3, -2)$.

(c) Emphasize the rewriting of equations such as $f(x) = 3x^2 - 1$ into the form $y = 3x^2 - 1$.

C. A suitable selection of problems might include 1 oral; 2b,c,f; 3a,c,d; 4c,f, and 5.

Problems 7 and 8 are for the more able student.

3.3 New Functions from Old Functions 2 periods

A. Objective: To investigate the graphical relation between the functions

$$y = f(x),\ y = f(x) + d,\ y = cf(x),\ y = cf(x) + d$$

B. Notes: (a) These investigations introduce the ideas of transformations as applied to graphing.

(b) The class may be divided into three groups with each group investigating one of the given functions. The results could be presented for each problem on the overhead.

(c) Begin the completion of the table for problem 1 as a group. This will get the students started on the right track.

(d) The mapping notation used in later transformation work may be introduced at this time.

i.e. $y = f(x) + d \qquad (x,y) \longrightarrow (x, y + d)$

$y = cf(x) \qquad (x,y) \longrightarrow (x, cy)$

$y = cf(x) + d \qquad (x,y) \longrightarrow (x, cy + d)$

This notation is convenient for future sketching.

C. Problems might include 1 (one function); 2a,b,f.

3.4 Relations 1 period

A. Objective: To define and graph relations.

B. Notes: (a) Define "Cartesian Product", $A \times B$ (read A cross B) as the set of all ordered pairs (x,y) where $x \in A$ and $y \in B$.

Note that most of the relations (the sets of ordered pairs) considered in this text are subsets of $R \times R$ - that is, the elements of both the domain and range must be real numbers.

(b) Note the vertical line test to determine if the graph of a relation represents a function.

(c) Rather than assigning the graphing in problem 6, the teacher may wish to cover the graphing of linear inequalities in Section 5.9.

C. Emphasize problems 1 to 5. Suggested graphing problems might be 7a,b,c,d,f,i.

3.5 Inverses of Relations and Functions — 1 period

A. Objective: To define and graph the inverse of a relation.

B. Notes: (a) Emphasize that the inverse of a function or relation is obtained by interchanging the components of the ordered pairs.

This idea can be extended to the equations of the sets as well. (See problem 7)

e.g. F: $y = x + 3$	interchange x and y $\longrightarrow$	F^{-1}: $x = y + 3$	
(1,4)		(4,1)	verify that these pairs
(-5,-2)		(-2,-5)	satisfy $x = y + 3$

(b) The concept of "reflection" in the line $y = x$ may be extended to introduce the mapping notation

$(x,y) \longrightarrow (y,x)$ (See Section 9.7 and summary on p. 292).

C. Discuss orally problems 1 to 3. Assign a selection of problems from 4 to 6. (In graphing problems, sketch the line $y = x$ to emphasize the reflection property.)

3.6 Applications

A. Objective: To sketch functions from everyday situations.

B. Notes: (a) In the sketches, show the dependent value on the vertical axis.

(b) Divide the class into groups to discuss the nature of the sketches for the assigned functions. Each group can prepare one sketch showing their opinion of the form of the graph. If there is disagreement when the sketches are posted, discussion should be continued.

Some guidelines for group discussion:

(i) Is the graph a straight line or curved?

(ii) Is their enough information to show any scale on the sketch?

(iii) Does the function increase or decrease or some of both?

(iv) How might one verify the proposed sketch?

(v) What other variables might affect the shape?

(vi) Are there maximum or minimum points?

(vii) Is there a specific domain or range for the graph?

Applications of linear functions are considered in detail in Section 4.4.

Chapter 4 <u>The Straight Line</u> 1 period

4.1 <u>Defining and Graphing the Straight Line</u>

A. Objective: To graph a straight line given its equation.

B. Notes: (a) The equation of a straight line is often referred to as a <u>linear</u> equation - the variable terms are of the first degree.

$$3x + 2y + 5 = 0$$ The exponent 1 is understood.

(b) A minimum of two ordered pairs is necessary, but a third provides a check that the points lie in a straight line.

(c) Stress that the intercepts are single values, not ordered pairs. e.g.

$$3x + 2y - 6 = 0$$ set $y = 0$, then $x = 2$

The intercept is 2, <u>not</u> (2,0).

(d) Emphasize the labelling of axes in all graph work.

C. Note problems 1g,h, 2f,g, 3,4,5, and 6, which refer to lines parallel to one of the axes. In problems 9i,j,k,l,m, and n, the student should simplify first.

Problems 1,2,3 in Exercise 2 of the Review and Preview to Chapter 4 may also be used here.

4.2 <u>Determining Equations of Lines</u> 2-3 periods

A. Objective: To determine the equation of a given line.

B. Notes: (a) This is essentially the beginning of the students work in analytic geometry. The basic idea is that <u>between any two points on a straight line, the slope is a constant</u>. Review the definition and formula for slope:

$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \text{ or } \frac{y_1 - y_2}{x_1 - x_2}$$

the order of subtraction must be the same for rise and run.

See Review and Preview Chapter 4, Exercise 1. This exercise including problems 1,4,5 and 6 may be a good preview for the exercise in this section

(b) The most used forms of the straight line equations are

(i) $y - y_1 = m(x - x_1)$ (ii) $y = mx + b$ (iii) $Ax + By + C = 0$

(c) Rearranging the form $Ax + By + C = 0$ to $y = -\frac{A}{B}x - \frac{C}{B}$ (that is in the form $y = mx + b$), the student should note that

$\boxed{m = -\frac{A}{B}}$. This idea is useful in determining the slope of $5x + 2y - 6 = 0$ as $m = -\frac{5}{2}$.

C. Problems 1,2,3 may be done orally. A substantial selection of problems should be assigned from questions 4 to 9 inclusive. Problem 10 reviews the idea that $f(5) = 7$ implies an ordered pair (5,7). Problems 11 and 12 use the forms $y = m(x - a)$ and $\frac{x}{a} + \frac{y}{b} = 1$ respectively. Students should note that both types can be done using the form $y - y_1 = m(x - x_1)$

i.e. (i) $m = 3$, $a = 4$ implies the point (4,0)

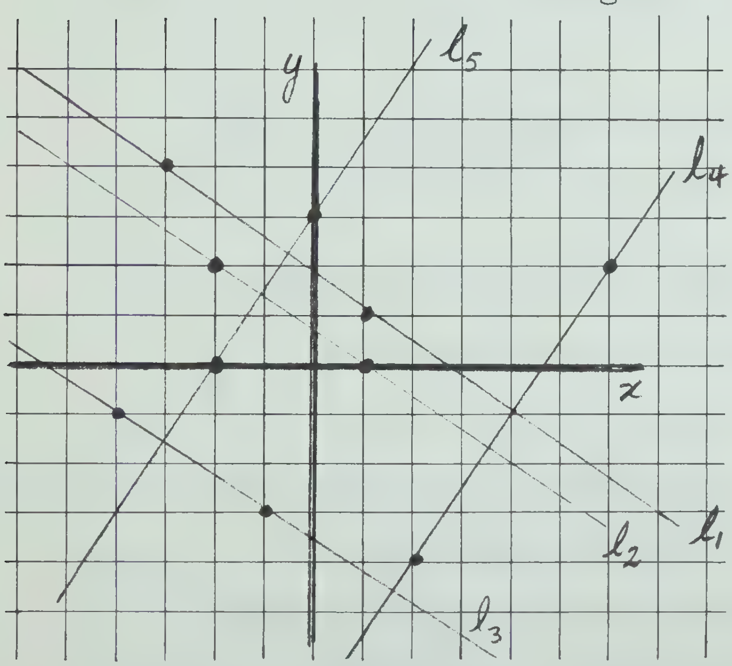
(ii) $a = 3$, $b = 7$ implies two points $(3,0)$ and $(0,7)$, and therefore the slope can be determined.

Problems 13 to 16 provide a challenge for the able student. Problems of this type are included in Exercise 4-3.

4.3 Parallel and Perpendicular Lines 2 periods

A. Objective: To apply slope concepts involving parallel and perpendicular lines.

B. Notes: (a) The proofs may be illustrated with numerical examples in order that the student may get on with the applications. Perhaps the more able student might be encouraged to work through the proof.



Calculating slopes and describing the relationships between the lines at left should illustrate the concepts

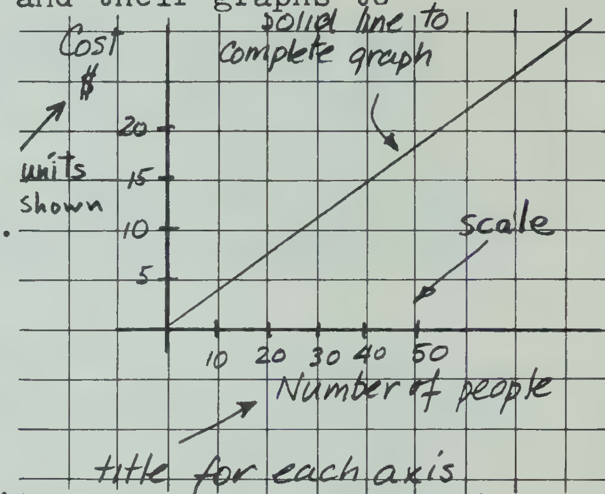
(i) $l_1 \parallel l_2 \iff m_1 = m_2$

(ii) $l_1 \perp l_4 \iff m_1 m_4 = -1$

C. Problems 1,2,3 may be done orally. Problems 5 and 6 should provide one assignment. Problems 7,8,9 introduce the idea of proofs using analytic methods. For the able student, a selection may be made from problems 10 to 14.

4.4 <u>Applications</u> 2-3 periods

A. Objective: To apply linear functions and their graphs to practical applications.



B. Notes: (a) In graphing, emphasize the points noted at right.

It is also a good idea to Title the entire graph.

(b) Before graphing, the limits of the problem should be established so that a suitable scale can be chosen for each axis. Two points:

(i) the larger the scale, the easier it is to read information.

(ii) the scale can be different for each axis, but it must be clearly indicated.

C. The lessons and assignments may be divided into three units:

(i) Example 1 followed by problems 1,2,4,10,12,13

Note that two pairs are given which allow the linear function to be graphed, and the slope to be calculated.

(ii) Example 2 followed by problems 3,5,6,7,8,9,11

Note that only problem 3 requires graphing. For all problems, determine the equation first using the method of Example 2(a)

(iii) Example 3 followed by problems 14 to 18.

This idea may be treated as an optional topic.

Chapter 5 Linear Systems

5.1 Systems of Linear Equations in Two Variables   2-3 periods

A. Objective: To solve linear systems in two variables.

B. Notes:

(a) Illustrate

(i) that a "multiple" of a linear equation is the same line.

e.g. $\left.\begin{aligned} 3x + y &= 5 \\ 6x + 2y &= 10 \end{aligned}\right\}$ are both satisfied by (2,-1) and (0,5)

(ii) that adding or subtracting equations l_1 and l_2, yields an equation, l_3, passing through the same point of intersection This idea is used in Algebraic Method 1.

e.g. $\left.\begin{aligned} 2x + y &= 5 \\ \underline{3x + 2y} &\underline{= 8} \end{aligned}\right\}$ intersect at (2,1)

$5x + 3y = 13$

also passes through (2,1).

(b) Emphasize that the "substitution" method should be used only when it is simple to isolate a variable in one equation.

e.g. $2x + 5y = 3$ ①

$x - 2y = 5$ ②

From ② $x = 5 + 2y$ is an easy step.

$2x + 5y = 3$ ①

$3x - 4y = 2$ ②

From ②, $y = \dfrac{3x - 2}{4}$ is not easy. Use method 1.

(c) The ideas in this section are important enough to warrant testing before proceeding to the applications.

C. Problem 1 provides an oral drill on the addition or subtraction method. In problems 4 and 5, the students are allowed to use the method of their choice, but should be encouraged to try both methods. If assigning parts of problem 7, illustrate first the idea of simplifying equations using "multiples".

e.g. $0.3x + 0.7y = 1$ becomes $3x + 7y = 10$ when multiplied by 10.

$\frac{x}{3} + \frac{y}{2} = 2$ becomes $2x + 3y = 12$ when both sides are multiplied by 6.

Problem 9 should only be assigned to the more able students.

5.2 <u>Applications of Linear Systems in Two Variables</u> 2-3 periods

A. Objective: To solve problems using linear systems in two variables.

B. Notes:

(a) A careful selection of the problem types is necessary. See Section C below for some suggestions.

(b) Stress the use of two variables even though it is possible to use only one.

(c) Emphasize the form of solution

e.g. Let x represent the ....
Let y represent the .... (include units)

$x + 2y = 5$ ①
$3x - y = 7$ ② *

* Since the students find this part to be the most difficult, perhaps assign only this much of each problem. After discussion, the students can complete the algebraic solution as a second step.

C. Suggested assignments:

(i) Problems 1 to 7 and #10 suitable for all students.

(ii) Problems 8,9 suitable after reviewing briefly the following: (a) 9% = 0.09

(b) $0.09x + 0.07y = 50$ becomes $9x + 7y = 5000$ when multiplied by 100.

(c) $I = Prt$

time (one year in these problems)
rate of interest
interest principal

(iii) Problems 11,12,13 are suitable if the first is discussed with the class as an example.

(iv) Problems 14 to 19 may be assigned using Examples 1 and 2 for reference.

(v) Problems 20 to 24 are suitable if the first one is discussed as an example.

5.3 Systems with Non-numerical Solutions

A. Objective: To solve systems of equations for non-numerical solutions.

B. Notes:

(a) Strictly an optional section for the able student, who may appreciate the solving of problem 2j to obtain a formula for the solving of linear systems:

i.e.
$$\left.\begin{aligned} x &= \frac{b_2c_1 - b_1c_2}{a_1b_2 - a_2b_1} \\ y &= \frac{a_2c_1 - a_1c_2}{a_2b_1 - a_1b_2} \end{aligned}\right\}$$

$$3x + y = 1$$
$$2x + 5y = 18$$

$$x = \frac{(5)(1) - (1)(18)}{(3)(5) - (2)(1)} = \frac{5 - 18}{15 - 2} = \frac{-13}{13} = -1$$

. . . .

(b) Reference to these types of problems is made on page 141, section 5.6.

5.4 Systems Of Linear Equations in Three or More Variables 1 period

A. Objective: To solve systems involving three or more variables.

B. Notes: (a) An optional topic, but a suitable challenge for the better student.

(b) The student should note that three equations are required if three variables are to be found.

5.5 Basic Matrix Operations 2 periods

A. Objective: To add and multiply matrices.

B. Notes: (a) The terminology and definitions of operations are clearly explained. Some practice is necessary to become competent with the "row-by-column" definition of matrix multiplication.

(b) The identity matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ occurs in many of the problems in future sections. Stress that this matrix has the same properties as the multiplicative identity, 1, in real number multiplication.

(c) The student should be made aware that equal matrices must have the same dimensions, and that $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ 5 & 6 \end{pmatrix}$ if and only if $a = 3$, $b = -1$, $c = 5$ and $d = 6$.

C. Note the difference between multiplication in problems 3 (the multiplication of a matrix by a real number) and 4 (the multiplication of matrices). Problem 5 provides some interesting thoughts for the more able student.

5.6 <u>Determinants and Inverses of 2 X 2 Matrices</u> 1 period

A. Objective: To find the determinant and inverse of a given 2×2 matrix.

B. Notes:

(a) To remember the determinant of $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ note the "cross products" $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

(b) Stress that the product of inverses is the identity element in real number multiplication also.

i.e. $\frac{3}{5} \times \frac{5}{3} = 1$

identity elements in multiplication.

$$\begin{pmatrix} 2 & -1 \\ 5 & -3 \end{pmatrix} \begin{pmatrix} 3 & -1 \\ 5 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

(c) Example 1 illustrates numerically the method of finding an inverse for a given matrix, while Example 2 (which requires the methods of Section 5.3) develops a formula.

If $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, the $M^{-1} = \begin{pmatrix} \frac{d}{ad - bc} & \frac{-b}{ad - bc} \\ \frac{-c}{ad - bc} & \frac{a}{ad - bc} \end{pmatrix}$

This step "factors" out the real number $\frac{1}{ad - bc}$.

this is det M $\quad = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

This illustrates that if M is to have an inverse, then det $M \neq 0$.

$$= \frac{1}{\det M} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Note: from $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, switch "a" and "d", and change the signs of "b" and "c".

C. Suitable problems should be selected.

5.7 <u>Solving Linear Systems Using Matrices</u> 1 period

A. Objective: To solve linear systems using matrices.

B. Notes: (a) In the introduction and Example 1, note the basic concept of multiplying both sides of an equation by the same thing - in this case the inverse of the coefficient matrix.

(b) As noted in the preamble to Example 2, the idea is to operate on the <u>rows</u> to obtain the row reduced echelon form,

$$\begin{pmatrix} 1 & 0 & 0 & c_1 \\ 0 & 1 & 0 & c_2 \\ 0 & 0 & 1 & c_3 \end{pmatrix}$$

which translates to the three equations

$$1x + 0y + 0z = c_1$$
$$0x + 1y + 0z = c_2$$
$$0x + 0y + 1z = c_3$$

which translates to the equivalent system $x = c_1$, $y = c_2$, $z = c_3$ to solve the problem.

(c) Note that steps 2 and 3 of the elementary row operations correspond to techniques used in solving systems by addition or subtraction.

(d) The method outlined in Example 2 is cumbersome, but, with practice and mental combination of steps, can be greatly abbreviated.

C. At this point, only a few of each type should be tried.

5.8 <u>Applications of Matrices</u>

A. Objective: To apply matrices to problem solving.

B. Notes: (a) The examples suggest that the section be separated into two units if covered:

(i) Example 1 and problems 1 to 3.

(ii) Example 2 and problems 4 to 7.

The students may enjoy the coding problems!

5.9 <u>Systems of Linear Inequalities in Two Variables</u> 1-2 periods

A. Objective: To graph systems of linear inequalities.

B. Notes: (a) This technique is particularly necessary if the following two sections on linear programming are to be covered.

(b) Encourage the use of intercepts in the graphing of the lines.

(c) The idea of rearranging the equations as in Example 2 must be practiced if it is to be used successfully.

(d) Before assigning problems, an example of the form $x \geq -5$ or $y < 3$ should be considered.

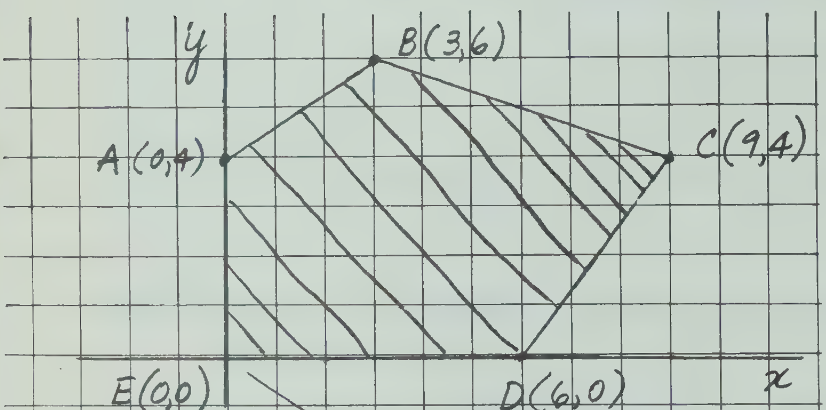
C. Oral discussion of problems 1 to 3. Assign a reasonable selection of parts from problems 4 to 6.

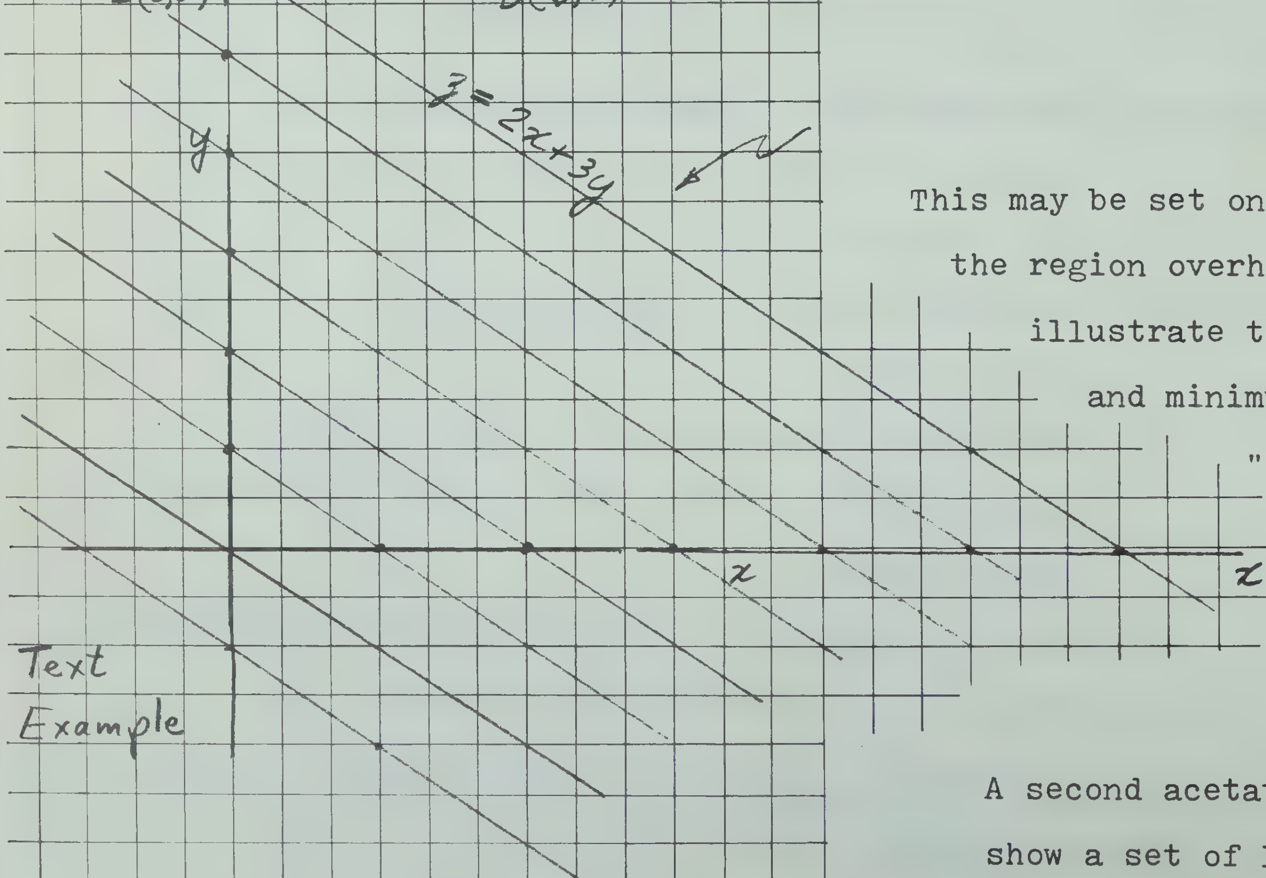
Problem 8 provides graphing which leads to the concepts of Section 5.10 and 5.11.

5.10 <u>The Vertex Theorem for Regions</u>   1 period

A. Objective: To illustrate and apply the vertex theorem for regions.

B. Notes: (a) The theorem can be illustrated using a region on an overhead, and several sets of parallel lines on separate acetates. The same scale should be used in all cases.





This may be set on top of the region overhead to illustrate the maximum and minimum cases for "z"

A second acetate could show a set of lines with slope $-\frac{1}{4}$.

C. Select suitable parts from problems 1 and 2. Problems 3,4, and 5 introduce the idea of graphing the region first.

5.11 Linear Programming 1-2 periods

A. Objective: To apply the vertex theorem for regions to linear programming problems.

B. Notes: (a) Complicated problems - suitable for the more able students.

(b) Emphasize the organizational chart which is established by careful reading of the problem.

(c) Once the inequalities are established, the graphing usually proceeds more easily. For any assigned problems, the first step should be to assign only the problem of determining these inequalities. The solving can be completed after these have been discussed and checked.

C. Select problems carefully - each one takes a fair length of time.

Chapter 6 Analytic Geometry

6.1 Midpoint of a Line Segment 1 period

A. Objective: To find the midpoint of a line segment.

B. Notes: (a) Emphasize the idea of "averaging" the coordinates to find the midpoint. One of the common errors is for students to use "rise" and "run", rather than addition of the coordinates. Note however that "rise" and "run" are used in development of the formula.

C. Problems 1,4,5,6 are straightforward applications. Perhaps examples of problems 2,3,7,8 will be necessary before assigning. Problems 8,9, and 10 may require a review of the slope ideas from section 4.3.

6.2 Area of a Triangle 1 period

A. Objective: To determine the area of a triangle given the coordinates of each vertex.

B. Notes: (a) This idea is optional, but will be necessary if the geometric ideas in section 6.4 are to be covered.

(b) The development in Example 2 looks more complicated than it really is. Note that it involves careful writing of subscripts - watch that students do not treat them as exponents. The method for remembering the formula is explained at the bottom of p. 169. Emphasize the listing of vertices in a <u>counterclockwise</u> direction to ensure a positive value. (or emphasize the use of absolute value bars).

(c) Example 4 extends the formula to any polygon by using the origin as one vertex for each triangle used. Perhaps this extension is suitable for the more able student.

C. Problems 2 and 3 should be assigned without explanation to allow students to discover that zero area indicates that the points are collinear. Ask them how else collinearity can be illustrated. (equal slopes - Review & Preview Chapter 4). Prior examples may be necessary before assigning problems 5 to 10.

6.3 Distance From a Point to a Line   1 period

A. Objective: To determine the distance from a given point to a given line.

B. Notes: (a) Stress that the distance from a point to a line is understood to be the perpendicular distance!

(b) Students at this level may find the development in Example 2 somewhat complicated. Perhaps an illustration of the formula in use would be sufficient at this time. Use the given information in Example 1 to verify the accuracy of the formula.

(c) Stress that the linear equation must be in the form $Ax + By + C = 0$ before the values of A, B, and C are determined.

C. Problems 1 and 2 provide practice in the use of the formula, while problems 3 and 4 will provide an additional challenge. Some discussion and examples are necessary before assigning problems 5 to 10.

6.4 Applications

A. Objective: To illustrate theorems in geometry using analytic methods.

B. Notes: (a) The problems in this section illustrate many ideas which the student will encounter in other studies in geometry. Emphasize that these problems do not constitute a "proof", since numerical coordinates are used.

(b) A review of formulas and definitions is essential before undertaking the problems.

Definitions: trapezoid, median, parallelogram, hypotenuse, diagonal, collinear, concurrent, altitudes.

Formulas: slope (R&P Chapter 4)
midpoint (Section 6.1)
area of a polygon (Section 6.2)
distance between points (R&P Chapter 6)
equations of straight lines (Section 4.2)
parallel and perpendicular lines (Section 4.3)

These could be reviewed in the discussion idea suggested below.

C. Problems should be selected carefully. A suggested approach might be to discuss each of the assigned problems with the class, noting the steps necessary to complete the problem. <u>Emphasize that a diagram should be made for all problems</u>.

e.g. Problem 4

(a) How can we show there is a right angle in $\triangle ABC$?

Possible answers:

(i) The diagram illustrates this fact. (Point out that this is not sufficient - but does show us which angle is probablу the right angle!)

(ii) Use Pythagorean Theorem. Ask which formula will be used.

(iii) Use slopes. Review formula, and the idea that the product must equal - 1 .

The student then completes the solutions discussed.

6.5 <u>The Circle</u> 1 period

A. Objective: To determine the equation of a circle with centre (0,0) and a given radius.

B. Notes: (a) The development of this formula is relatively straightforward.

(b) The students could be encouraged to discover the ideas developed in Example 3 by selecting test points inside and outside the circle (as in Section 5.9).

C. Problems 1 to 4 may be covered orally. An example may be necessary prior to assigning problem 5. A compass should be used in graphing the circles in problems 6 and 7. Problems 8 to 10 will challenge the able student.

Chapter 7 Congruence and Parallelism

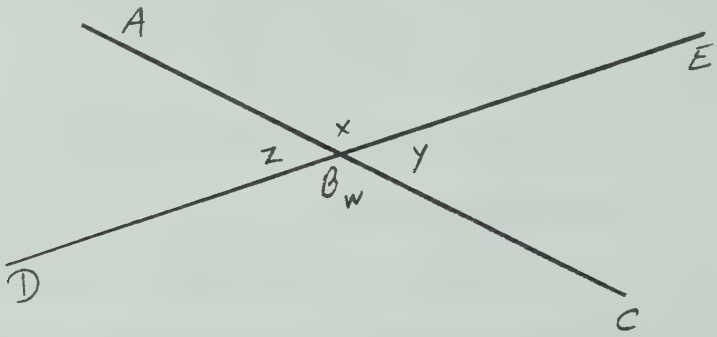
7.1 Angle and Congruence Theorems 2-3 periods

A. Objective: To prove a variety of deductions using angle and congruence theorems.

B. Notes: (a) This may be the student's first exposure to deductive geometry. If this is the case, a careful and slow beginning may prevent some future problems.

(b) The proof of the theorems OAT, CAT, SAT may be used to illustrate the methods of this section.

e.g. OAT



Proof: (i)	(ii)	Reason
$x + y = 180^\circ$	$\angle ABE + \angle EBC = 180$	straight angle
$x + z = 180^\circ$	$\angle ABE + \angle ABD = 180$	straight angle
$x + y = x + z$	$\angle ABE + \angle EBC = \angle ABE + \angle ABD$	both equal to 180°
$y = z$	$\angle EBC = \angle ABD$	subtraction

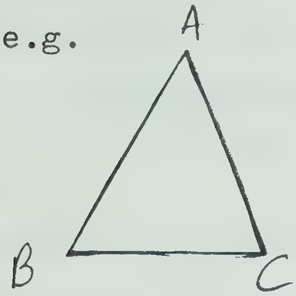
Proof (i) illustrates the idea, while proof (ii) is the form to be used in this chapter. Note that the "reasons" can be definitions, theorems, or <u>brief</u> explanations, but should be stated for each line of the proof.

(c) Note that the congruence postulates are simply <u>stated</u>, without a proof being required.

(d) In using congruency, emphasize that the triangles should be named before listing the appropriate inequalities.

(e) Stress that ITT may be used either way:

e.g.



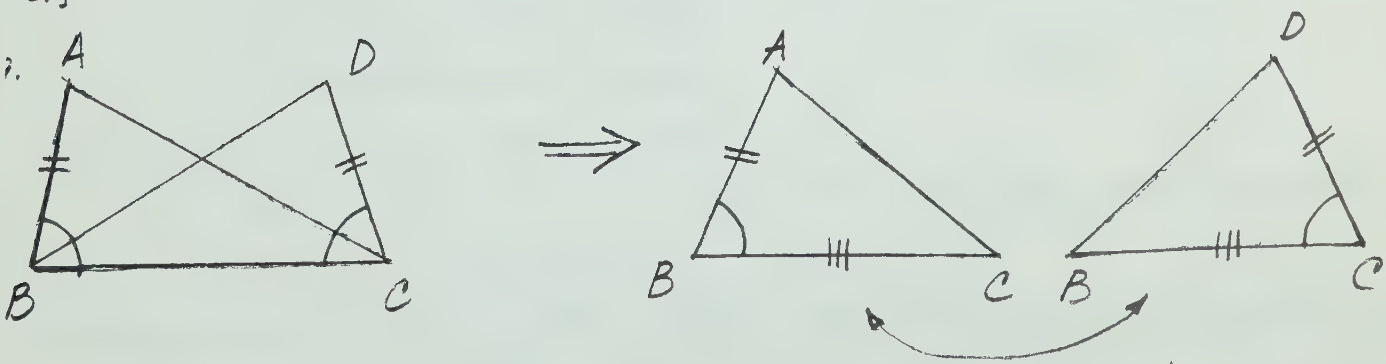
(i) If AB = AC, then $\angle B = \angle C$
(ii) If $\angle B = \angle C$, then AB = AC

ITT

Each statement is a <u>converse</u> of the other, and ITT is a <u>dual</u> theorem.

C. Problem 1 is suitable for oral practice. Have the student name the pairs of corresponding equal angles and sides. In overlapping triangles (problems 3,8), the student may wish to separate the triangles for clarification.

e.g.



Note: common side, and therefore equal.

In problems 11 to 15, stress the four parts of a complete deductive solution: given, required, diagram, and proof.

7.2 Inequality Relations of a Triangle — 2 periods

A. Objective: To prove and use the inequality relations of a triangle.

B. Notes: (a) The ideas in this section are challenging to many students, and thus many teachers treat this as an optional section. The concepts can be illustrated inductively by measuring sides and angles of given triangles.

(b) The steps in an indirect proof are stated at the top of p. 190. Perhaps an algebraic problem could be used to illustrate the steps.

Examples.

(1) Given $x, y, z \in R$, and $x \neq y$, prove that $x + z = y + z$

Proof: (i) Either $x + z = y + z$ or $x + z \neq y + z$

(ii) Assume $x + z = y + z$ since this is the negation of what we want to prove.

(iii) Since $x + z = y + z$

$\therefore \quad x = y$ subtraction

but $x \neq y$ is given and therefore our assumption is false.

(iv) $\therefore \quad x + z \neq y + z$

(2) Given a,b,c are positive real numbers and $a \neq b$,

prove $\dfrac{a + c}{b + c} \neq \dfrac{a}{b}$

Proof: Either $\dfrac{a + c}{b + c} = \dfrac{a}{b}$ or $\dfrac{a + c}{b + c} \neq \dfrac{a}{b}$

Assume $\dfrac{a + c}{b + c} = \dfrac{a}{b}$

$\therefore \quad b(a + c) = a(b + c)$ multiplication

$$ba + bc = ab + ac \qquad \text{distributive}$$

$$\therefore \quad bc = ac \qquad \text{subtraction}$$

$$\therefore \quad b = a \qquad \text{division}$$

but $b \neq a$ is given, and therefore the assumption is false.

Hence, $\frac{a + c}{b + c} \neq \frac{a}{b}$

(c) The method of indirect is needed in section 7.3 but can be introduced and used at that time.

C. Problems 1 to 4 are suitable for oral practice. Problems 5,6,7 should provide sufficient practice for the average student.

7.3 <u>Parallel Lines</u> 3 periods

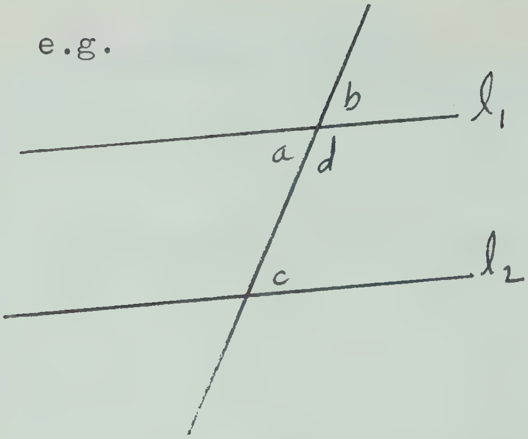
A. Objective: To prove deductions using the Transversal Parallel Theorem.

B. Notes: (a) Note that in defining parallel lines, it is agreed that "a line is parallel to itself".

(b) Stress that a transversal can meet two lines whether the lines are parallel or not. (See problem 2 of the exercise)

(c) The student should be encouraged to use the diagrams illustrated on p. 194 when using TPT, and should also note the converse statements involved in TPT.

e.g.



(i) If $l_1 \parallel l_2$, then $a = c$ and

$b = c$ and

$c + d = 180°$.

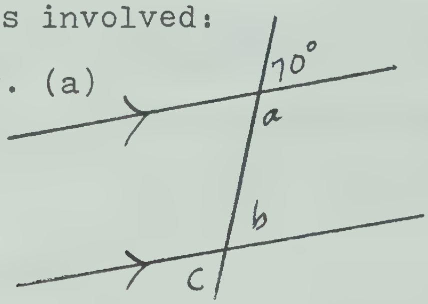
(ii) if $a = c$ or

$b = c$ or

$c + d = 180°$, then $l_1 \parallel l_2$.

C. Problems 1 and 2 provide practice in the basic use of TPT. Encourage the students to be specific in naming the type of angles involved:

e.g. (a)



$b = 70°$ corresponding angles (TPT)

and $c = 70°$, $a = 110°$

A suggestion in the assigning of deductions is to present the problems using an overhead, discuss possible methods of proof (marking up the diagrams as you go), and then assign the proofs to be written correctly for homework. Three or four deductions are usually sufficient as an assignment.

7.4 <u>Parallelograms and Trapezoids</u> 2 periods

A. Objective: To prove deductions involving parallelograms and trapezoids.

B. Notes: (a) The theorems in this section are essentially deductions which the students can prove themselves. Problems 1,2,3 can be assigned immediately, and the results summarized after completion.

(b) The theorem QPT on p. 198 resembles the solution used in problem 10 of Exercise 7-3.

(c) The theorems TMT and MPT should be discussed with the students since certain constructions are required to initiate the proofs.

C. Selected problems from 1 to 10 should provide sufficient practice. Problems 5 and 6 should be combined, since one method to prove problem 6 is to extend CB and DA to meet at E which creates a diagram resembling number 5. A hint to "Join AC" might be given for starting problem 10. Problems 11 and 13 are somewhat challenging, and do require a complete solution from the student.

7.5 <u>Figures Between the Same Parallels</u> 2-3 periods

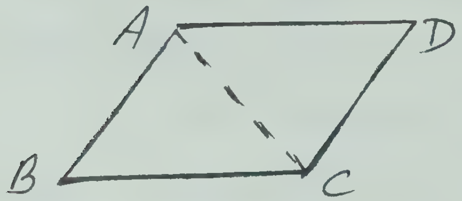
A. Objective: To solve problems involving figures between the same parallel lines.

B. Notes: (a) Stress the facts that (i) congruent triangles have equal area by definition, denoted by the statement;

$\triangle ABC = \triangle DEF$

implies equal area

and (ii) that since the diagonal of a parallelogram ABCD bisects the area;



$\triangle ABC = \triangle ADC = \frac{1}{2}$ //gm ABCD.

(b) The student should memorize the formulas on p. 204.

C. Problems 1 to 4 use the formulas and the theorems in numerical problems - useful as a first assignment.

Problems 6 and 7 may be treated as theorems and used in later deductions.

Once again a discussion of possible methods of solution before assigning various problems will prevent some problems.

e.g. #6 Hint: Through P draw a line parallel to AB and CD.

#7 Student should note that //gmABCD and //gmEBCF are between the same parallels, as are //gmGHCF and //gmEBCF

7.6 The Theorem of Pythagoras 2 periods

A. Objective: To prove and use the Pythagorean Theorem and its converse.

B. Notes:

(a) It is sufficient to discuss the steps of the proof with the class as a whole. The emphasis should be on the use of the theorem in numerical problems and selected deductions.

(b) In proving the converse (on p. 208), the student should note that the Pythagorean Theorem can be used in $\triangle XYZ$;

$\therefore \; ZY^2 = ZX^2 + XY^2$ and since $ZX = AC$, $XY = AB$, and $BC^2 = AB^2 + AC^2$, it can be shown that $BC = ZY$.

(c) A thorough discussion of Example 2 should provide the student with some ideas which may be used in the exercise.

C. In problem 2, the form of solution must be watched.

e.g. 2(b)

$$2^2 + \sqrt{3}^2 = 4^2$$

this statement is incorrect.

$$4 + 3 = 16$$

Encourage a left side - right side format.

L.S. $2^2 + \sqrt{3}^2$	RS 4^2	
$= 4 + 3$	$= 16$	
$= 7$		then conclude the triangle is not right angled.

In problem 6, show that $(m^2 + n^2)^2 = (m^2 - n^2)^2 + (2mn)^2$ using the above form.

In problems 7,8,9,12,13, encourage the student to write the possible Pythagorean statements, and then attempt to use the appropriate ones to prove what is required.

Problem 15 should be assigned, and emphasized as another useful congruency method.

7.7 <u>Angles of a Polygon</u> 1 period

A. Objective: To develop and apply the formula for the sum of the interior angles in a polygon.

B. Notes: (a) Stress the definition of a regular polygon as one with all sides and angles equal.

C. All problems are suitable for most classes. Note that problems 3,4, and 5 suggest that the sum of a set of exterior angles for any polygon is $360°$.

7.8 <u>Applications in Three Dimensions</u>

A. Objective: To apply geometric concepts to problems in three dimensions.

B. Notes: (a) It is worthwhile to apply the volume and lateral area formulas illustrated in Example 1 to problems 5 and 6.

C. The geometry problems should be previewed by some discussion. Example 2 suggests ideas for problems 8, 9, and 10. Problem 7. Join BD. Use postulate II (ii) p.214. In many of the other problems, use the concept that if two lines are parallel to the same line, then they are parallel to each other.

Chapter 8 Similarity

8.1 Ratio and Proportion 2 periods

A. Objective: To define and apply the properties of proportions.

B. Notes: (a) Note that in the ratio a:b, that $b \neq 0$.

(b) The student should become familiar with the properties summarized on p. 227. This familiarity will facilitate the later work in geometry.

C. Use problem 1 to emphasize the basic properties of proportions. Problem 2 is best handled using property (a) - note the use of the distributive property in many parts. In problem 3, emphasize the fact that the proportion $\frac{x}{15} = \frac{14}{y} = \frac{2}{5}$ yields three distinct proportions.

(i) $\frac{x}{15} = \frac{14}{y}$ (ii) $\frac{14}{y} = \frac{2}{5}$ (iii) $\frac{x}{15} = \frac{2}{5}$

Proportions (ii) and (iii) are the most useful in the solution of this problem. This idea is used in Example 2 and 3 in the following section.

For problems 8 and 9, the form of solution may resemble

$$x + y = 50 \quad (1)$$

$$\frac{x}{y} = \frac{3}{7} \quad (2)$$

Solve the two equations,

However at this time, you may wish to introduce the "K method" illustrated in example 4, section 8.2. This method may also be useful in solving problems 11 and 13.

8.2 Multiple Ratios   1 period

A. Objective: To solve problems involving ratios of three or more terms.

B. Notes: (a) Refer to the notes on section 8.1.

(b) The "K method" can be used in problems such as Example 1, but it is more cumbersome. The concept however is perhaps worth illustrating along with the method shown.

e.g.

$\frac{x}{3} = k$,	$\frac{7}{y} = k$,	$\frac{2}{8} = k$
$x = 3k$	$7 = ky$	$\therefore 8k = 2$
$\therefore x = 3 \times \frac{1}{4}$	$\therefore 7 = \frac{1}{4} y$	$k = \frac{1}{4}$ ← step 1
$x = \frac{3}{4}$	$28 = y$	

(c) Solutions should include opening statements for word problems as illustrated in Examples 3 and 4.

C. Selected problems from 1 to 8. The "k method" solves problem 13 nicely. e.g.

$$a = bk, \quad c = dk, \quad e = fk$$

$$\text{LS} : \frac{bk + dk + fk}{b + d + f}$$

simplifies to k which equals the right side.

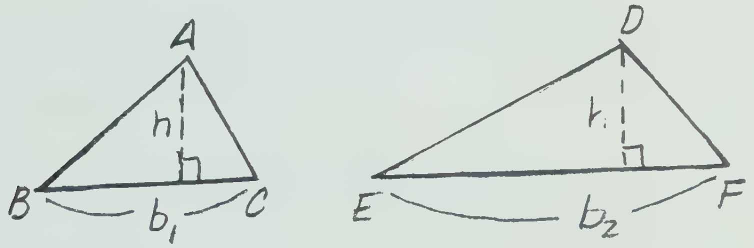
8.3 The Division of Sides Theorem   2 periods

A. Objective: To use the Division of Sides Theorem (DST) and its converse in calculating lengths and providing parallelism .

B. Notes: (a) The theorem DST is used in the proof of the theorems in sections 8.4 and 8.5.

(b) Before proving the theorem, establish the concept that "if two triangles have the same altitude, the ratio of their areas is equal to the ratio of their bases."

e.g.



$$\frac{\triangle ABC}{\triangle DEF} = \frac{\frac{1}{2} b_1 h}{\frac{1}{2} b_2 h} = \frac{b_1}{b_2}$$

(c) The diagram and concepts illustrated in Example 3 will also appear in section 9.8 involving Dilatations.

C. The exercise could be divided into two sections: problems 1 to 4, and problems 5 to 10.

The "Angle Bisector Theorem" should be assigned to the more able student since this concept appears in mathematics contest problems.

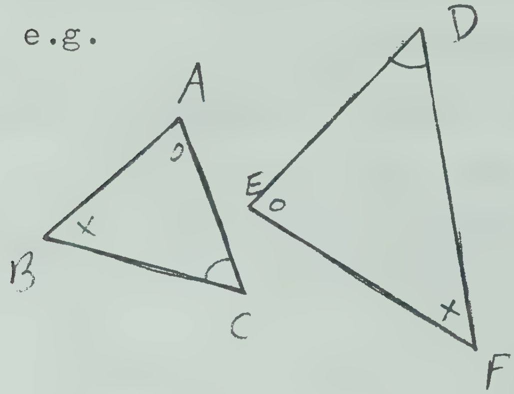
8.4 Similar Triangles 2 periods

A. Objective: To define similar triangles, and to prove and use the (AA) Theorem.

B. Notes: (a) Note the relationship between similar and congruent triangles: in congruency, the ratios are 1:1.

(b) When writing the similarity statement, the student should state corresponding vertices in the same order.

e.g.



$\triangle ABC \sim \triangle EFD$ or $\triangle ABC$ $\sim \triangle EFD$

The writing of the proportions is made more obvious.

i.e. $\frac{AB}{EF} = \frac{BC}{FD} = \frac{AC}{ED}$

(c) Emphasize the corollary of the theorem, i.e. (AA$\sim$) as the form to be used in problems.

(d) In Example 3, students may be advised to substitute first in the proportion, then solve.

(e) The student should be able to manipulate statements such as $MP \cdot PA = SP \cdot PR$ into appropriate proportions:

e.g.

If $ab = cd$, then $\frac{a}{c} = \frac{d}{b}$ or $\frac{c}{a} = \frac{b}{d}$ or $\frac{a}{d} = \frac{c}{b}$ . . . .

C. Problems 1,2, and 3 may be done orally. The overlapping triangles in problem 5 sometimes cause confusion. Perhaps separate the triangles, noting the common angle S in both. This idea also occurs in future sections.

In assigning selected problems from 6 to 13, it may be advisable to discuss diagrams and methods before the student writes complete solutions.

8.5 <u>More Similarity Theorems</u> 1-2 periods

A. Objective: To prove and apply the (SAS$\sim$) and (SSS$\sim$) theorems.

B. Notes: (a) Note that the method of proof is similar to that used for (AA$\sim$).

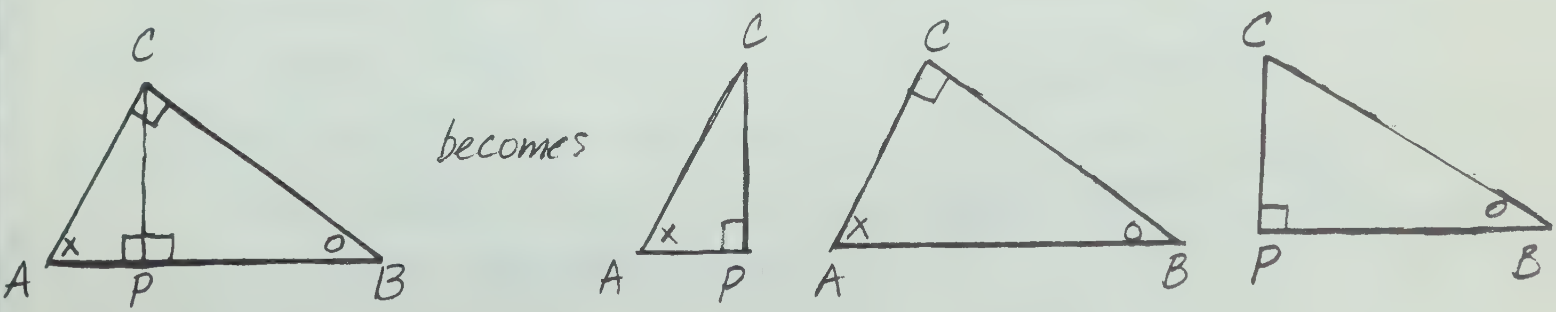
C. Assign selected problems to reinforce the theorems. Note that problems 7 and 8 require the manipulation of equations into corresponding proportions. (refer to Example 1)

8.6 <u>Mean Proportional in a Right Triangle</u> 1 period

A. Objective: To prove and apply the Mean Proportional Theorem.

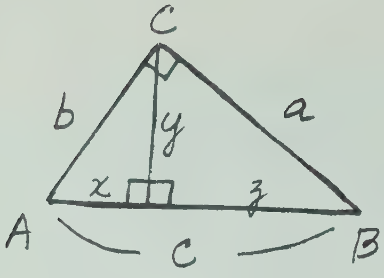
B. Notes: (a) This section deals with the geometric interpretation of a mean proportional. Review the concept that "x" is the mean proportional between "a" and "b" if and only if $\frac{a}{x} = \frac{x}{b}$. (see section 8.1)

(b) Figure 8-4 may be separated if necessary to clearly show the similar triangles:



(c) Students should construct a memory - aid diagram for the theorem and its corollaries.

then $\frac{x}{y} = \frac{y}{z}$ and $\frac{x}{b} = \frac{b}{c}$ and $\frac{z}{a} = \frac{a}{c}$



(d) The construction of a mean proportional may be an optional exercise.

C. Emphasize problems 1 to 3 to reinforce the theorem.

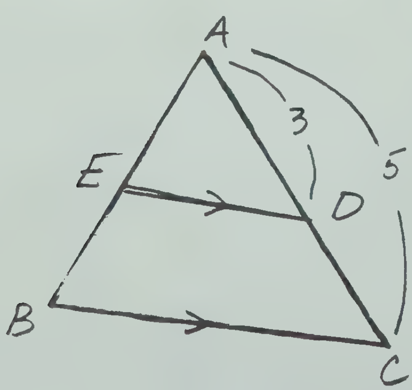
8.7 <u>Areas and Volumes of Similar Figures</u>   2 periods

A. Objective: To develop and apply the relationships between areas and volumes of similar figures.

B. Notes: (a) If the section is covered, the emphasis should be on the ratio of areas of plane figures.

C. Emphasize problems 1,3,4,5,7, and 8 as numerical examples.

Note problem 7:



(a) $\frac{\triangle AED}{\triangle ABC} = \frac{9}{25}$ ← from this statement, the area of $\triangle$ABC is 25 square units if $\triangle$AED is 9 square units.
$\therefore$ trapezoid EBCD must be 16 square units.

(b) $\therefore \quad \frac{\text{quad DEBC}}{\triangle \text{ABC}} = \frac{16}{25}$

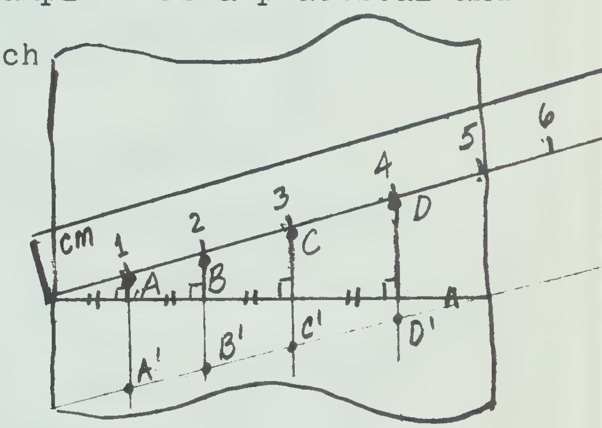
This idea may be used in problem 8 also.

8.8 Division of a Line Segment 2 periods

A. Objective: To divide a line segment internally or externally.

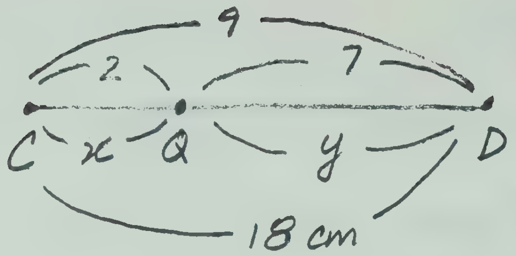
B. Notes: (a) Emphasize the summary shown in the box on p. 251. The idea shown may be illustrated using the form AP:PB = a:b which shows the initial point, the point of division and the terminal point in order.

(b) The construction method of example 2 is a practical and useful illustration. The sketch at right shows how a piece of material can be divided into 5 equal widths very quickly. A second position of the ruler will yield points A',B',C',D' which may be joined to A,B,C,D respectively.



(c) The methods illustrated in Examples 3 and 4 have useful applications in the students mathematical future. Problem 13 develops a formula for internal division.

C. Problems 1 to 5 should be assigned first. Stress the idea of sketching the problem first, marking on the given units, and then setting up the solution.



ratio units

actual measurements

Then the student can set up the proportions, and select the necessary ones to solve.

$$\frac{x}{y} = \frac{2}{7}\ , \ \frac{x}{18} = \frac{2}{9}\ , \ \frac{18}{y} = \frac{9}{7}\ , \ \text{etc.}$$

actual ratio

The construction (problems 6,7,8) may be included in the first assignment.

A second assignment could emphasize selected parts of problems 9 and 10. The more able student should try 11, 12, and 13.

8.9 Applications

A. Objective: To apply the concepts of similar triangles to typical applications.

B. Notes: (a) As illustrated in Example 1, the student should label the triangles, and show why they are similar.

(b) Stress the use of concluding statements (with units) for problems of this type.

C. Emphasize problems 1 to 6. Problems 7,8,9 depend on the importance given to Section 8.7.

Chapter 9 Vectors and Transformations

9.1 Vectors 1 period

A. Objective: To define vectors geometrically and algebraically.

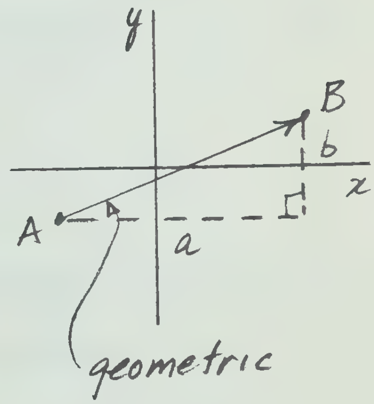
B. Notes: (a) Review (i) formula for slope

(ii) the fact that parallel lines have the same slope.

(b) Stress the various methods of representing a vector:

$$\vec{v} = \overrightarrow{AB} = [a,b]$$

initial point terminal point algebraic



In [a,b], "a" is the run (+ to right, - to left), and "b" is the rise (+ up, - down)

$\therefore$ slope of the vector is $m = \frac{b}{a}$ and magnitude $|\vec{v}| = \sqrt{a^2 + b^2}$

(c) Equal vectors must (i) have equal magnitude

(ii) be parallel (same slope)

(iii) have the same direction.

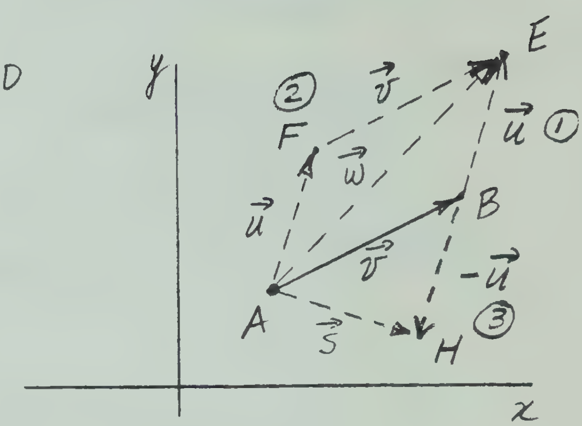
(d) In Example 1, note that for $\vec{v} = [-4,3]$, if (x,y) is the initial point, the $(x - 4, y + 3)$ is the terminal point. This leads to the mapping $(x,y) \longrightarrow (x - 4, y + 3)$ which represents a translation.

C. Do problems 1 to 4 orally. Assign problems 5 to 9.

9.2 <u>Addition and Subtraction of Vectors</u> 2 periods

A. Objective: To add and subtract vectors.

B. Notes: (a) Using the Triangle Law, the remaining concepts of vector addition and subtaction can be illustrated using vectors such as $\vec{u} = \overrightarrow{CD} = [1,3]$ and $\vec{v} = \overrightarrow{AB} = [4,2]$ at right.



1 Construct $\vec{u}$ at B terminating at E.

By definition $\overrightarrow{AB} + \overrightarrow{BE} = \overrightarrow{AE}$

or $\vec{v} + \vec{u} = \vec{w}$ i.e. $[4,2] + [1,3] = [5,5]$

2 Add $\vec{u} + \vec{v}$ starting at A. Vector $\vec{u}$ terminates at F, and when $\vec{v}$ is constructed, the terminal point is also E. This illustrates the commutative property. Also, since quad ABEF can be shown to be a parallelogram, the sum of $\vec{u}$ and $\vec{v}$ is the <u>diagonal</u> of this parallelogram.

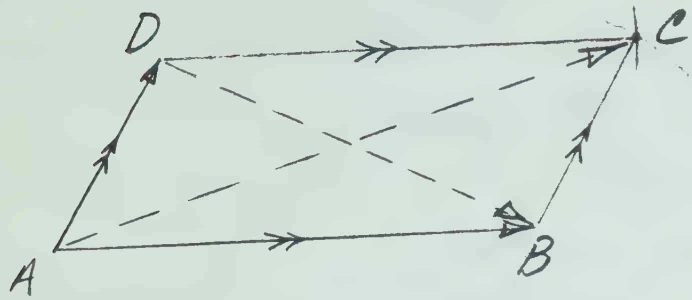
3 The opposite of $\vec{u} = [1,3]$ is $-\vec{u} = [-1,-3]$. Construct $-\vec{u}$ starting at B and terminating at H.

Then $\overrightarrow{AB} + \overrightarrow{BH} = \overrightarrow{AH}$ <u>or</u> $\vec{v} + (-\vec{u}) = \vec{v} - \vec{u}$

$= [4,2] - [1,3] = [3,-1]$

Note also that $\overrightarrow{AH} = \overrightarrow{FB}$, and hence the //gm ABEF can be used for adding and subtracting vectors.

(b) The $\parallel$gm Law is more convenient when adding or subtracting geometric vectors without graph paper.



See construction p. 181

$\overrightarrow{AB} + \overrightarrow{AD} = \overrightarrow{AC}$

and $\overrightarrow{AB} - \overrightarrow{AD} = \overrightarrow{DB}$

C. Oral problems 1 to 5. In problems 7 and 8, emphasize the Parallelogram Law.

If discussing problem 14, it is convenient to use a transparent rectangular solid.

If assigned, problems 15, 16, 17 should be discussed previously.

i.e. #15 Assign $\vec{u} = [a,b]$, $\vec{v} = [c,d]$ $a,b,c,d \in R$

$\vec{u} + \vec{v} = [a,b] + [c,d]$

$= [a+c,\ b+d]$ definition

$= [c+a,\ d+b]$ commutative property of real numbers (not vectors)

$= [c,d] + [a,b]$ definition

9.3 <u>Multiplication of a Vector by a Scalar</u> 1 period

A. Objective: To define multiplication of a vector by a scalar.

B. Notes: (a) A <u>scalar</u> has no direction - it is simply a real number.

(b) Stress that (i) the length of a vector is affected by multiplying by a scalar, k.

(c) Provide an example of the type

$2[-5,3] + 3[1,-2]$ before assigning exercise.

$= [-10,6] + [3,-6]$

$= [-7,\ 0]$

C. Oral problems 1,2. Assign 3,6 (a diagram is necessary. Also recall the facts about parallelograms)

Proof of problem 4 is necessary if Section 9.4 is to be covered.

e.g. Let $\vec{u} = [a,b]$ and $\vec{v} = [c,d]$

$LS = k([a,b] + [c,d])$

$= k[a+c,\ b+d]$ definition

$= [k(a+c),\ k(b+d)]$ definition

$= [ka+kc,\ kb+kd]$ distributive for real numbers (not vectors)

$= [ka,kb] + [kc,kd]$ definition

$= k[a,b] + k[c,d]$ definition

$= k\vec{u} + k\vec{v}$

9.4 <u>Geometry with Vectors</u> 2 periods

A. Objective: To prove deductions using vector concepts.

B. Notes: (a) This concept requires time to develop, but can be useful.

(b) Stress that if $\vec{AB} = k\vec{CB}$, then $AB \parallel CD$ and $AB = kCD$, and <u>to prove $AB \parallel CD$, $AB = kCD$, we must prove $\vec{AB} = k\vec{CD}$</u>.

(c) As vectors are introduced in the solution, direction arrows should be marked on the diagram!

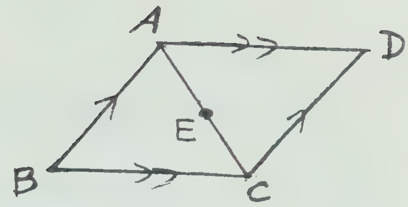
C. Problem hints.

1. Join AC. Show $\vec{AD} = \vec{BC}$.
2. Show $\vec{AD} = \vec{BC}$ then use problem 1.
3. Join BD. Use the hint to show $\vec{FG} = \vec{EH}$. Then use problem 1.
4. Show $\vec{BY} = \vec{XD}$ after proving $\vec{AY} = \vec{XC}$.
5. Show $\vec{PQ} = \frac{1}{2}(\vec{AD} + \vec{BC})$.
6. Assume E is midpoint of AC.

Show $\overrightarrow{DE} = \overrightarrow{EB}$

which shows DEB is a

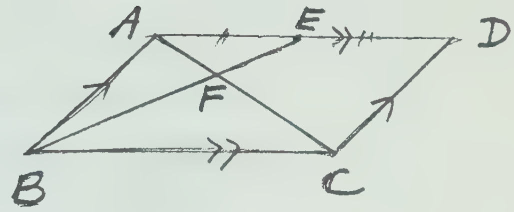
straight line.



7. Show $\overrightarrow{DE} = \frac{k}{k+1}\overrightarrow{BC}$.

8. Show $\overrightarrow{BF} = 2\overrightarrow{FE}$

$\overrightarrow{CF} = 2\overrightarrow{FA}$.



9.5 <u>Translations</u> 1 period

A. Objective: To define and apply translations.

B. Notes: (a) The properties can be illustrated using Example 1. To show image lines parallel, use slopes.

(b) Note the relationship between a vector [a,b] and the translation $T:(x,y) \longrightarrow (x+a, y+b)$

C. Oral problems 1,2. Perhaps not necessary to assign all of 3, and both 4 and 5.

Note that in 7 and 8, the values "a" and "b" cancel when calculating length and slope for $A'(x_1 + a, y_1 + b)$ and $B'(x_2 + a, y_2 + b)$.

Problem 9 should pose a challenge - perhaps suggest graphing the line and its image first.

9.6 <u>Rotations and Composition</u> 2 periods

A. Objective: (i) To define and apply rotations.

(ii) To define and apply the compositions of transformations.

B)notes: (a) Note the properties for rotations are the same as for translations, except for parallelism.

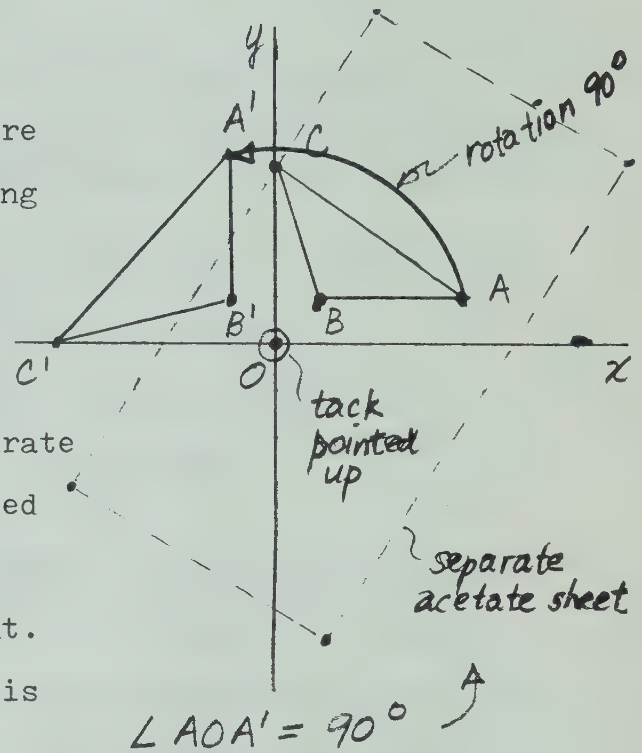
(b) It is assumed that a rotation of 90^0 (which is $+90^0$) is counterclockwise. Negative angles indicate a clockwise rotation.

(c) This section should be separated into two lessons:

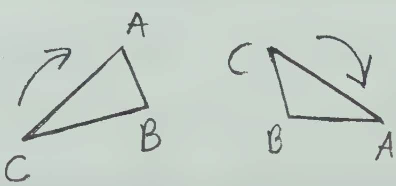
(i) rotations

(ii) composition

(d) A compass and protractor are needed in problems involving rotation. To illustrate rotations (with or without graph paper), the original figure is copied on a separate piece of acetate and rotated with the tack holding the centre of rotation constant.



Note that the sense of $\triangle ABC$ is preserved



(e) Emphasize that in composition of transformations $X \circ Y$ the transformation Y occurs first!

As shown in Fig. 9-8, p.278, composition is most easily illustrated on graph paper. (Use of the overhead is very much desired)

C) Oral problems 1,2

Assignment (i) Problems 5,6,7,10,12

(ii) Problems 4,8,9

In problem 5, develop the mappings for 90°, 180° and 270° using the P(x,y) row. (See 292 for summary) Use these in problem 6.

Note

The teacher may restrict the discussion of this section to those problems involving graphical solutions only. These concepts are more easily discussed and illustrated.

9.7 Reflections and Symmetry 1-2 periods.

A) Objective: To define and apply reflections.

B) Notes: (a) Once again the concepts can easily be developed using a graphical approach. The student should be encouraged to learn the mappings on p.281 for application in the problems.

(b) Note Example 2 relating two reflections to a rotation:

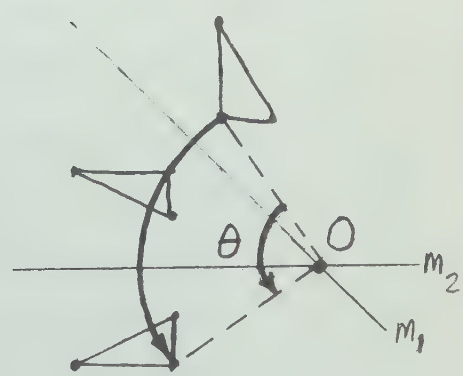
(i) rotation of 180° and

$(x,y) \to (-x,-y)$ (ii) reflection in one axis followed by reflection in the other axis.

The concept illustrated here may be generalized: "Two reflections are equivalent to a rotation if the mirror lines intersect." The point of intersectic is the centre of rotation.

Paper folding and pins can be used to demonstrate the correct idea.

Have the students discover the relationship if the mirror lines are parallel! (a translation)



C) Oral problems 1-4

Problems 5,11,14 could use paper folding or a transparent mirror

Assign problems 6,7,8 (The discussion of results of 8 (c) should lead to the mapping $(x,y) \rightarrow (y,x)$ as a reflection in $y = x$. See also Section 3.5, p.91 regarding inverses of relations)

Problem 13 is a good summary of ideas. A graph should be used.

9.8 Dilatations 2 periods

A) Objective: To apply dilatations to geometric figures.

B) Notes: (a) Review DST, Section 8.3; Similar triangles, section 8.4

(b) Note Example 1 which develops the mapping $(x,y) \rightarrow (kx,ky)$ for a dilatation with magnification factor k, and centre (0,0). This allows the use of analytic formulas for slope and length to show the parallelism and proportionality of lengths.

(c) Example 2 (a) may be expanded to

(i) show $K'M' \parallel KM$ (using slopes)

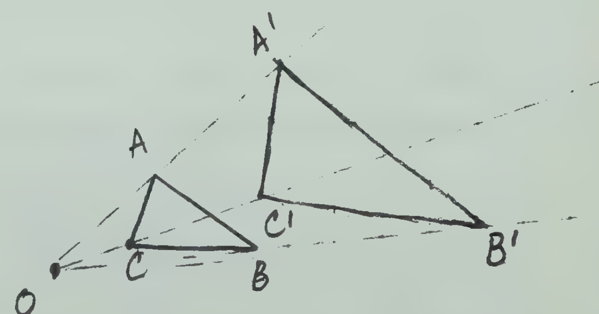
(ii) show $K'M' : KM = 1:2$ (using lengths)

(iii) show $\triangle K'L'M' \sim \triangle KLM$ (using SSS$\sim$)

(iv) show $\triangle K'L'M' : \triangle KLM = 1:4$ (using AST p.248)

C) Oral problems 1,2,3,5

For problem 4(a):(i) draw lines through OA, OB, OC as shown.



(ii) Since OA:OA' = AB:A'B' (see proof in this section), construct OA' = 4 OA using a compass. Similarly construct OC' = 4 OC and OB'=4OB

Join A'B' and C'

Problems 6,7,9 are useful for the graphical approach.

9.9 Transformations using Matrices   Optional

A) Objective: To express rotations, reflections, and dilatations as 2×2 matrices.

B) Notes: (a) If covered previously, review matrix multiplication in Section 5.5.

(b) Note the method of Example 2 which may be applied to the solution of problem 4

C) Emphasize problems 1,2, and 3. Problems 4 and 5 relate to Examples 2 and 3.

Note the summary of Transformations on p.292.

Chapter 10 <u>The Circle</u>

10.1 The Parts of a Circle 1-2 periods

A) Objective: (i) To define the parts of a circle.

B) Notes: (a) The terminology should be discussed thoroughly so that the student may readily recall definitions throughout the chapter.

C) Problems 2,3,4,7,8,9 should be emphasized since the results can be used later. Deductive methods are used in these proofs.

The proofs for problems 10,11,12,and 13 will depend on the emphasis given to rotations in Section 9.6. The results however should be noted, and problems 14,15, and 16 attempted.

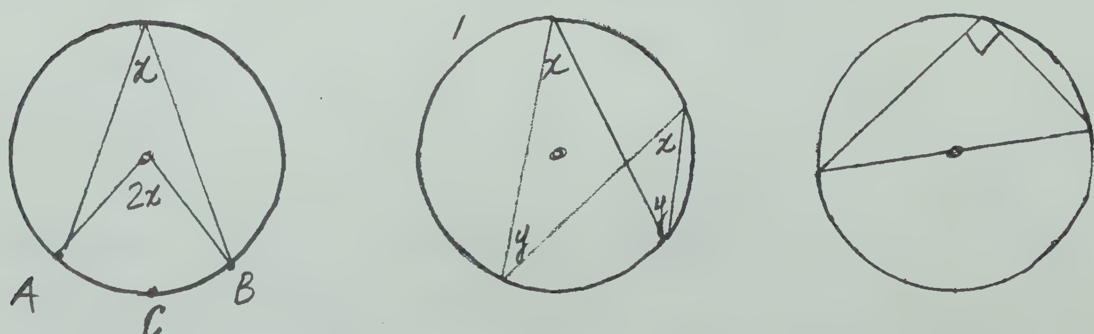
10.2 Inscribed Angles 2-3 periods.

A) Objective: To prove and apply the Angles in a Circle Theorem (ACT).

B) Notes: (a) An overhead showing cases 1,2, and 3 is helpful in discussing the proof of ACT.

(b) Corollaries 1 and 2 have many applications and should be emphasized.

(c) Construct memory aid diagrams for ACT

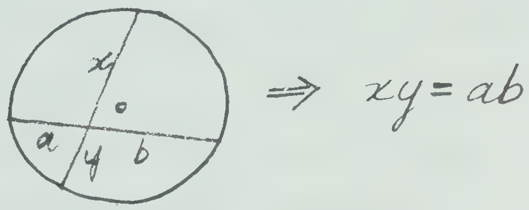


to accurately specify an arc, three letters can be used . ie. arc ACB

C) Problems 1 and 2 (plus additional numerical problems on an overhead) should be stressed to reinforce the idea of angles subtended by the same arc, as well as the concepts of ACT.

Students should use a compass in constructing all circle diagrams!

The student should be reminded of (AA$\sim$) when problems 7 and 8 are assigned. The result of problem 8 is worth illustrating for future use.



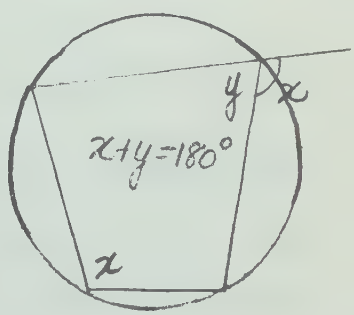
Problems should be selected from numbers 6,9,10,11,12,13, leaving 14,15,16 for the able student.

10.3 Cyclic Quadrilaterals 1-2 periods

A) Objective: To prove and apply the Cyclic Quadrilateral Theorem.

B) Notes: (a) An overhead illustrating cases 1,2, and 3 allows the indirect proof to be discussed quickly with the class.

(b) A memory diagram should be drawn for the theorem and corollary at the top of p.309.

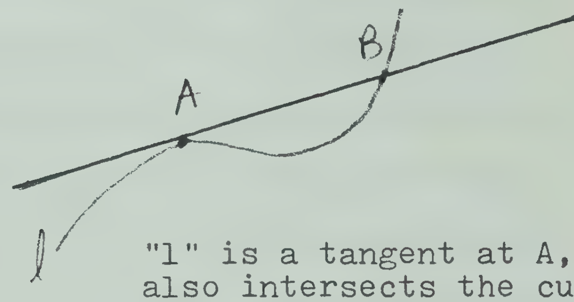


C) Problems 1 to 4 reinforce the concepts in the theorem (CQT). Select problems from 5 to 13 suitable for the ability of the class.

10.4 Tangents 1-2 periods

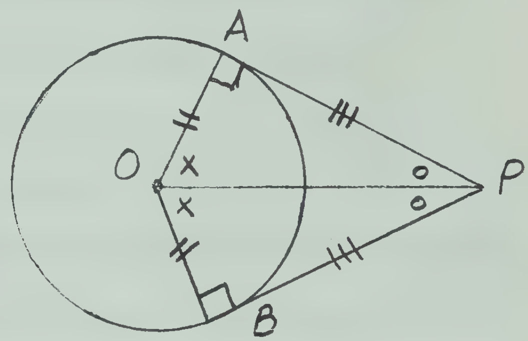
A) Objective: To prove and apply the Tangent Theorem.

B) Notes: (a) Point out that the tangent definition on p. 311 is for circles but not for more complicated curves.



"l" is a tangent at A, but also intersects the curve at B.

(b) A diagram should summarize the Tangent Theorem



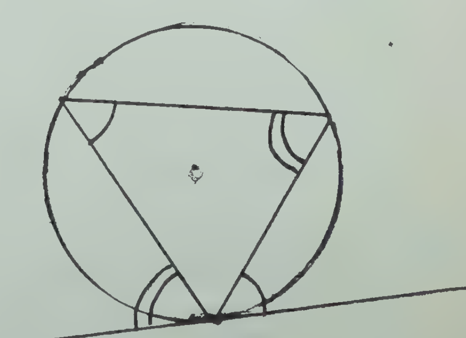
C) Select suitable problems from 1 to 12. Note that the concept of equal tangent segments (PA = PB above) is used in many problems!

10.5 The Tangent Chord Theorem 2 periods

A) Objective: To prove and apply the Tangent Chord Theorem

B) Notes: (a) In locating, the related angles, students must remember the inscribed angle is on the opposite side of the chord, and begins and ends at the end points of the chord.

Note: in each diagram there are <u>two</u> pairs of equal angles!



C) Problem 1 provides a review of the circle theorems to this point as well as the tangent chord theorem. Select suitable problems from 2 to 8.

Problem 9 develops the Tangent Secant Theorem which is applicable in the solutions for problems 9 to 13.

10.6 Equations of Tangents 1 period

A) Objective: To determine the equation of a tangent to a circle and, the length of a tangent segment.

B) Notes: (a) This is an analytic geometry application of the Tangent Theorem (a).

(b) Review the relationship between slopes of perpendicular lines, and the forms of linear equations (Sections 4.3 and 4.2 respectively)

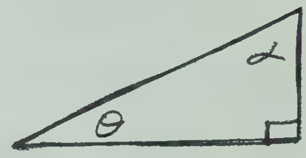
C) Assign problems 1 and 2. Note that parts 1f and 2f develop formulas for these problems. The student may note the pattern in the equations before this point.

Chapter 11 Trigonometry

11.1 Trigonometric Ratios of an Acute Angle. 1-2 periods

A) Objective: To define the six trigonometric ratios in terms of the sides of a right angled triangle.

B) Notes: (a) Stress that the opposite and adjacent sides depend on which acute angle is being considered.



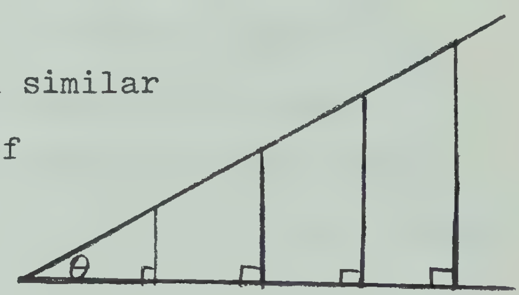
(b) To remember the primary ratios try

"SOHCAHTOA"

$\text{Sine}\,\theta = \frac{O}{H}$ $\text{Cosine}\,\theta = \frac{A}{H}$ $\text{Tan}\,\theta = \frac{O}{A}$

* Note that since H is the longest side, $\sin\theta$ and $\cos\theta$ will always be less than or equal to 1.

(c) Stress the idea that from similar triangle work, the ratios of all pairs of corresponding sides in the triangles at right will be equal. Hence the ratios for θ do not depend on the length of the sides.

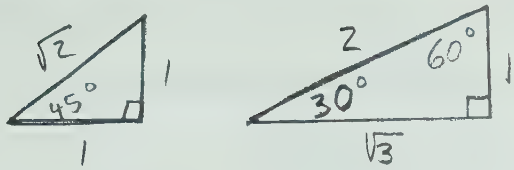


C) Assign problems 1 to 7 with perhaps an oral discussion of 8. Problems 9 and 10 may be optional.

11.2 Trigonometric Ratios of Special Angles 1 period

A) Objective: To determine the trigonometric ratios for 45°, 30°, and 60°.

B) Notes: (a) The student should memorize the diagrams on p.329:



Refer to Exercise 7-6, problem 3 on p.209

C) Assign selected problems from 1 to 10

Note $\sin^2\theta = (\sin\theta)^2 = (\sin\theta)(\sin\theta)$

11.3 Trigonometric Ratios of Any Acute Angle 1 period

A) Objective: To find the trigonometric ratios for $0^\circ \leq \theta \leq 90^\circ$.

B) Notes: (a) Briefly discuss the reciprocal relationships

(b) Stress that care must be taken to ensure that the correct value is read from tables! Use of a straight edge is advised to avoid jumping rows.

(c) Note that the tables provide approximate values for trig. ratios, whereas in the section 11.2, for example, $\cos 45^\circ$ is exactly $\frac{1}{\sqrt{2}}$.

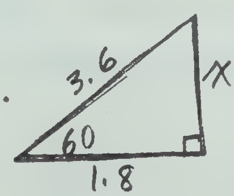
C) Assign all problems.

11.4 Solution of Right Triangles 2 periods

A) Objective: To solve right triangles.

B) Notes: (a) Illustrate that trigonometric solutions usually involve simpler computation than the Pythagorean Theorem.

e.g.



(ii) $\tan 60^\circ = \frac{x}{1.8}$

(i) $x^2+1.8^2=3.6^2$

$x^2=12.96-3.24$

$=9.72$

$x = \sqrt{9.72}$

$\doteq 3.1$

simple multiplication → $x \doteq 1.8 \times 1.732$

$x \doteq 3.1$

Note: (i) the use of ≐ symbol since the ratios used are not exact.

(ii) it is conventional to express answers to the same accuracy as the given values.

(b) Stress also that the form

$\boxed{\frac{\text{unknown}}{\text{known}} = \text{trig. ratio}}$ leads to simpler computation.

This is the reason why all six ratios should be known.

(c) Assign sufficient problems from 1 to 5 for the student to develop competence in this type of question.

Test trigonometric concepts at this point.

11.5 Applications of Right Triangles 2-3 periods

A) Objective: To solve problems involving trigonometric concepts and right triangles.

B) Notes: (a) The ideas in Examples 1 and 2 provide concepts applicable to problems 1 to 13.

(b) Emphasize (i) neat, labelled diagrams

(ii) correct form of solution

(c) Examples 3 and 4 illustrate more difficult concepts applicable to problems 14 to 20. These types may be optional, since the use of the Sine Law and Cosine Law in Chapter 12 will permit alternative, and perhaps more straightforward, solutions.

C) See notes above for problem assignment.

Chapter 12 Trigonometric Functions

12.1 Trigonometric Ratios of Any Angle 1-2 periods

A) Objective: To determine the trigonometric ratios for any angle in standard position.

B) Notes: (a) Emphasize that coordinates x and y can be positive or negative, but that "r" is always positive. See Example 3.

(b) Emphasize that these definitions are for angles in standard position only.

C) Problem 1 may be done orally. Radicals should be expressed in lowest terms ie. 1(a) $\sqrt{52} = 2\sqrt{13} = r$

Assign selected problems from 2 to 6. Problem 7 should challenge the more able student.

12.2 Use of Tables for Angles Greater than 90° 1 period

A) Objective: To determine trigonometric ratios for angles greater than 90°.

B) Notes: (a) Rather than memorize the relationships proven in this section, the student may wish to note:

(i) the angle θ involved is always between the terminal arm and the x-axis.

(ii) the ratio is always the same ie. $\sin(180^\circ + \theta) = -\sin\theta$.

(iii) the sign may be determined by the CAST rule.

Note the CAST rule may be used for the reciprocal ratio signs also

C cosine and secant

A all

S sine and cosecant

T tangent and cotangent.

S	A
T	C

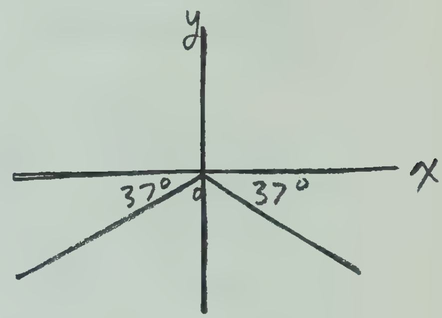
C) Assign all problems

Before assigning 3, it may be necessary to provide an example.

eg. Solve $\sin\theta = -0.6018$

Solution:

Since $\sin\theta$ is negative, θ lies in quadrant 3 or 4.



From tables
$0.6018 = \sin 37^\circ$

$= 180^\circ + 37^\circ$
$= 217^\circ$ or $= 360^\circ - 37^\circ$
$= 323^\circ$

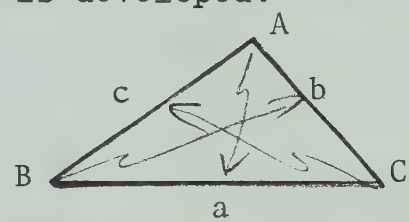
12.3 The Law of Sines 1-2 periods

A) Objective: To develop and apply the Law of Sines.

B) Notes: (a) Note that the proof involves the right triangle definitions.

See that it is clear to the students that the perpendicular height, "h", used in the proof is not necessary after the formula is developed.

(b) Emphasize the convention for labelling sides of a triangle



(c) Examples 1,2, and 3 illustrate cases where the Sine Law is applicable.

Discuss the SSS and SAS with the students as an introduction to the need for another law(the Cosine Law in Section 12.4).

(d) Note the case SSA, although correct in Example 3 can create problems at times. For a complete discussion of this "ambiguous case", see FMT-Senior, Section 6.4, p.191. (by the same authors).

(e) Note that the two forms of the Sine Law allow the unknown to always appear in the numerator.

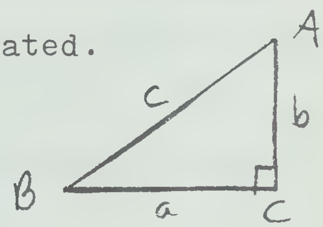
C) Selected problems from 1 to 3. Remind students that "solving" a triangle means determining all unknown sides and angles. Encourage the inclusion of diagrams in a solution!

12.4 The Law of Cosines 1-2 periods

A) Objective: To develop and apply the Law of Cosines

B) Notes: (a) Illustrate that the Sine Law does not work for the SSS and SAS cases.

(b) See that students note the pattern in the law so that it is necessary to memorize one form only, from which the others can be stated.



(c) Note that the cosine law results in the Pythagorean statement.

e.g. $$\begin{aligned} c^2 &= a^2+b^2-2ab\cos 90^\circ \\ &= a^2+b^2-2ab(0) \\ &= a^2+b^2 \end{aligned}$$

Ask the students why this may not be used as a proof of the Pythagorean Theorem (Surely it's easier than p.207) - cannot be used since the theorem was used to develop the law!

(d) Be prepared if someone comments on the examples, that if $\sin A \doteq 0.7402$, then $\angle A \doteq 48^\circ$ or 132° Remind them that $\angle A + \angle B + \angle C = 180^\circ$.

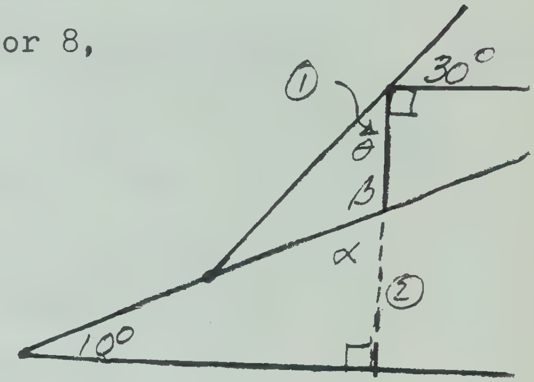
C) Assign selected problems from 1 to 4.

12.5 Applications of Oblique Triangles 1-2 periods

A) Objective: To apply the Sine and Cosine Laws to problems involving oblique triangles.

B) Notes: (a) Emphasize the use of a diagram for each problem

(b) Before assigning problem 3 or 8, a discussion of diagrams of that form might be advantageous to decide how necessary angles may be determined.



$\theta = 60^\circ$

$\alpha = 180^\circ - (90^\circ + 10^\circ)$

$= 80^\circ$

$\therefore \beta = 100^\circ$

C) Assign problems 1 to 10

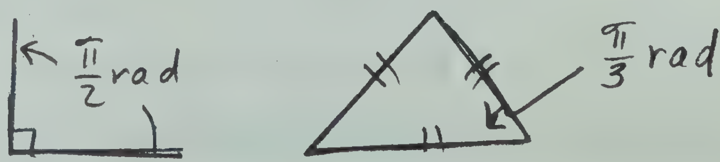
12.6 Radian Measure 1 period

A) Objective: To convert degree measure to radian measure, and vice versa.

B) Notes: (a) Stress that radian measure is useful in problems involving arc length of a circle (See Example 4)

(b) The student should become familiar with the size of angles for common radian measures.

e.g.



C) Assign selected problems from 1 to 6.

12.7 Graphs of the Trigonometric Functions 2 periods

A) Objective: To graph the sine, cosine, and tangent functions.

B) Notes: (a) It is probably sufficient to use one of the methods (i.e. the table) to generate the graphs of the $\sin\theta$, $\cos\theta$, and $\tan\theta$ functions. Also the use of the trigonometric tables may save time in completing the tables for $0 \leq \theta \leq 2\pi$. This would require completing only parts a, b, and c for each investigation.

12.8 Trigonometric Equations 1 period

A) Objective: To solve trigonometric equations

B) Notes: (a) See the notes on Section 12.2. The graphs of Section 12.7 could be used to verify approximately some of the results of Exercise 12.2, problem 3.

C) Problems may be discussed with the class as a group using overheads of the trigonometric function graphs.

Chapter 13 Probability and Statistics

13.1 Mathematical Probability 1 period

A) Objective: To define the mathematical probability of an event.

B) Notes: (a) Students will probably have intuitive ideas of the probability of an event.

(b) Emphasize that the sample space is the set of all possible outcomes.

C) Oral discussion of problems 1,2,3.

Problem 5 - the sample space is $\{S,S,S,T,T,T,I,I,A,C\}$

i,e, the repetitions must be included. Also in problem 6.

All problems should be assigned and discussed.

13.2 Empirical Probabilities 1-2 periods

A) Objective: To consider a specific example illustrating empirical probability.

B) Notes: (a) Prior to discussing Example 1 illustrate the difference between mathematical (theoretical) probability and empirical probability.

e.g. (i) Flip two coins. Record the results.

After only 4 flips, has exactly one pair of heads occurred? After 8 flips, exactly two pairs?

Outcome	tally	total
HH		
HT		
TH		
TT		

After many flips, calculate

$$P(H,H) = \frac{\text{number of times HH has occurred}}{\text{total number of flips}}$$

Is this result exactly equal to $\frac{1}{4}$ or (0.25)?
Other simple experiments (rolling dice, drawing cards, etc) will illustrate further that empirical probability may be used to make reasonable predictions after many events.

Also note the experiment suggested in problem 4 of the exercise.

C) Use the exercise after a discussion of Example 1.

13.3 Mutually Exclusive Events 1 period

A) Objective: To find the probability of mutually exclusive events.

B) Notes: (a) Note the use of the Venn diagram in Example 2 to illustrate the fact that the events are not mutually exclusive.

(b) Note the relationship to the union of the two events.

C) Use problem 1 for oral discussion. A suggestion might be to discuss the remaining problems to decide on which events are mutually exclusive - then assign the problems.

13.4 Independent Events 1 period

A) Objective: To find the probability of independent events.

B) Notes: (a) A suggestion might be to split the section and exercise into two parts:

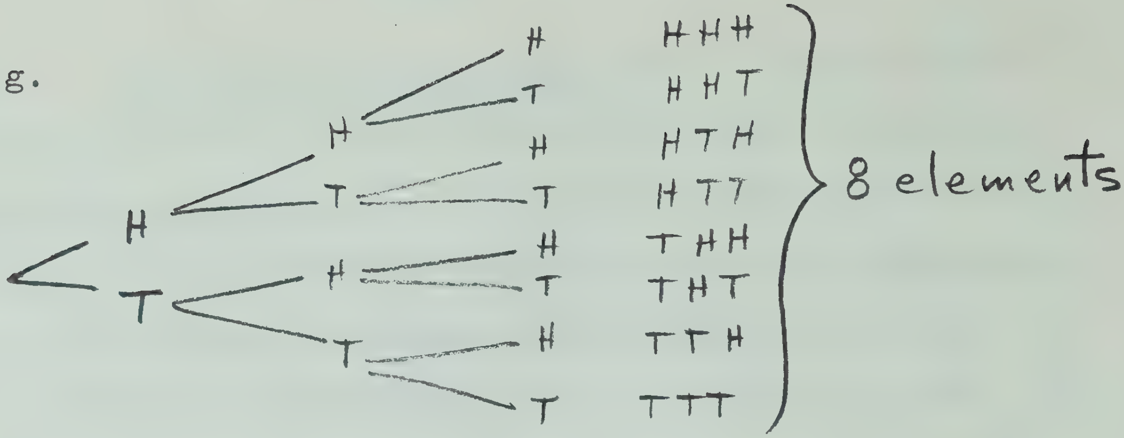
(i) Example 1, followed by problems 1 to 6.

(ii) Example 2, followed by problems 7 to 10.

(b) Note the relationship to the intersection of the two events.

C) Problem 1 usually provides an interesting discussion. If the student uses the sample space $\{HHH, HHT, HTT, TTT\}$, the probability looks like $\frac{1}{4}$. However, the use of a tree diagram can clarify the problem.

e.g.



Problems 11 and 12 provide a challenge for the more able student.

13.5 Statistics Optional.

A) Objective: To define statistics and discuss sampling techniques.

B) Notes: (a) Have students bring in articles from magazines, newspapers, etc., where the concepts of statistics and sampling have been used. These can be used as a basis for the discussion of sampling techniques and their validity.

C) Use the problems for discussion of sampling techniques and their validity.

13.6 Frequency Distributions 2 periods

A) Objective : To prepare histograms and frequency polygons from a frequency distribution table.

B) Notes: (a) Encourage the students to perform their own "experiments" to obtain the raw data:

i.e. roll of a single die
roll of a pair of dice
marks from their last class test
traffic samples

(b) Emphasize that the histogram and frequency polygon present the data in a clear, easily analyzed form.

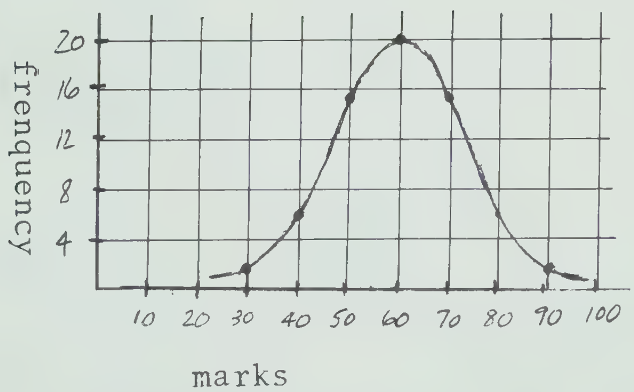
C) Assign selected problems or have students use their own data to prepare histograms.

13.7 Measures of Central Tendency 1 period

A) Objective: To determine the mean, median, and mode for given data.

B) Notes: (a) Use a table such as the one below to produce the curve as shown. This introduces the bell shaped normal curve which is used in discussions of statistical analysis.

Test marks	frequency
10	0
20	0
30	1
40	6
50	15
60	20
70	15
80	6
90	1
100	0



Note: (i) such a curve usually appears only after the graphing of a very large number of results. (ii) The mean, median, and mode are equal for this curve.

As a comparison, construct a curve for the last set of class test results. Calculate the mean, median, and mode.

(b) Emphasize that the total must be known in calculating the mean(see Example 1).

C) Use problem 1 for discussion of the suitability of using mean, median, or mode. Ask for other sample situations for each measure.

13.8 Measures of Dispersion 2-3 periods

A) Objective: To determine the range and standard deviation for a set of data.

B) Notes: (a) Emphasize that standard deviation indicates how the data is spread out with respect to the mean.

i.e. a large s.d. implies the data is widely spread, while a small s.d. implies the data is clustered around the mean.

(b) In problems, the normal curve is used, but the horizontal scale is not added until the standard deviation is computed. (See p. 388 and Example 2)

C) Select suitable problems from 1 to 9.

Answers to Puzzle Problems

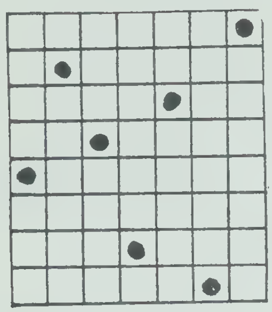
p. 4 74 min

p. 6 a=4
b=2
c=8
d=5
e=7

p. 9
$$\begin{array}{r} 9332 \\ 402 \\ 852 \\ \hline 10586 \end{array}$$

p. 11 28

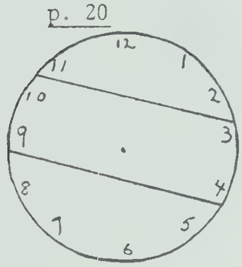
p. 13



p. 15 22 games

p. 17 1806

p. 20



p. 23 37 games

p. 25 answer varies

p. 28 start

9	9	5	5	3	3	3	3	3
5	0	0	4	4	5	1	1	3
4	0	4	0	0	0	4	3	3
2	0	0	0	2	1	1	2	0

p. 29 milestone

p. 31 flour
floor
flood
blood
brood
broad
bread

p. 33 7744

p. 37
$$\begin{array}{r} 09809 \\ 12\overline{)1089708} \\ \underline{108} \\ 97 \\ \underline{96} \\ 108 \\ \underline{108} \\ 0 \end{array}$$

p. 38 $18^2 + 5^2 + 4^2 + 1^2$

p. 43 Dec. 30

p. 44 9 times the original area.

p. 48
$$\begin{array}{r} 9635 \\ 546 \\ 185 \\ \hline 10366 \end{array}$$

p. 55 16

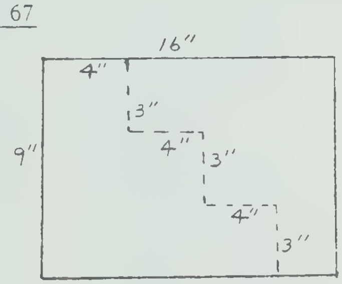
p. 56 4 - 2 + 8 = 10
9 + 6 - 5 = 10
3 ÷ 1 + 7 = 10

p. 59 Dreamt

p. 61 16 - 4

p. 63 one
owe
ewe
eye
rye
roe
toe
top
two

p. 67



p. 70 1681

p. 74 9 ÷ 3 + 6 = 9
5 x 1 + 4 = 9
2 x 8 - 7 = 9

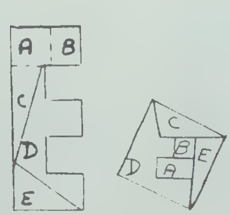
p. 78 $54

p. 80 Mike and Sue
Allen and Carol
Ed and Jane

p. 83 bookkeeper

p. 90 A vs. C 4 to 2   B vs. A 3 to 0   B vs. C 0 to 0

p. 95



p. 102 comb
come
home
hole
hale
hall
hail
hair

p. 104 Alex 40 h
Ed 60 h

p. 106
$$\begin{array}{r} 246 \\ \underline{591} \\ 837 \end{array}$$

p. 110
Into the boat since the weight is displaced.

p. 116 (i) split 3,3,2; weigh 3's
(a) If they balance, weigh the two to find the lighter one.
(b) If they don't balance, split lighter 3 into 2 and 1. Weigh the 2; if they balance the 1 is the lighter one.

p. 118 pity
pits
pots
lots
loss
lose
hose
home
some
sole
sold
gold
good

p. 125 42

p. 129 4 - 6 ÷ 2 = 1
9 - 8 x 1 = 1
5 + 3 - 7 = 1

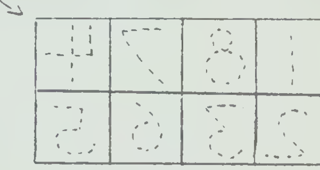
p. 132

8L	5L	3L
8	0	0
5	0	3
5	3	0
2	3	3
2	5	1
7	0	1
7	1	0
4	1	3
4	4	0

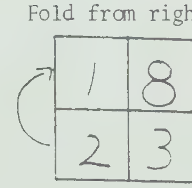
p. 135 x = 4
y - 2

p. 138

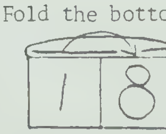
Place the paper face down



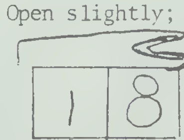
Fold from right to left.



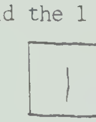
Fold the bottom under.



Open slightly; fold the 4 and 5 over the 6.



Fold the 1 and 2 over the 3.



p. 143 Man

p. 145
12 - 0 - 0
5 - 7 - 0
5 - 2 - 5
5 - 5 - 2
10 - 0 - 2
3 - 7 - 2
8 - 4 - 0
8 - 0 - 4
1 - 7 - 4
1 - 6 - 5
6 - 6 - 0

p. 147

1) $L_1H_1 \longrightarrow L_1$, $\xleftarrow{H_1}$

2) $H_1H_2 \longrightarrow H_1$, $\xleftarrow{H_2}$

3) $H_2L_2 \longrightarrow L_2$, $\xleftarrow{H_2}$

4) $H_2H_3 \longrightarrow H_2$, $\xleftarrow{H_3}$

5) $H_3L_3 \longrightarrow H_3L_3$

p. 153
bread
break
bleak
bleat
blest
blast
boast
toast

p. 159
2178
4
8712

p. 167

p. 171
rest
lest
lost
loft
soft
sofa

p. 176
W - BC
O - T
E - M
C - R

p. 185 Sovereignty

p. 189
tiger
tiler
tiles
toles
roles
roses

p. 190 8 pairs

p. 194
251453
60753
312206

p. 220
$\dot{3} \times 1 + 5 = 8$
$9 - 7 + 6 = 8$
$8 \div 2 + 4 = 8$

p. 226 2 m

p. 237 In the answer, the first 2 digits are the age, the second 2 digits are the change.

$\underbrace{100(10x+y)}_{\text{age}} + \underbrace{(10a+b)}_{\text{change}}$

p. 249 0, 1

p. 254
hare
hard
lard
laid
said
sail
soil
soul
soup

p. 261

p. 264
x=3
y=2

p. 271
1x2+5=7
9÷3+4=7
8+6-7=7

p. 274 1953125 and 512

p. 276
eye
lye
lie
lid

p. 280 adventitions

p. 288 60 min

p. 289 72.56

p. 291
32x46-23x64
34x86=43x68
39x31=93x13
82x14=28x41

p. 292 $101 - 10^2 = 1$

p. 301
763
2884
3647

p. 305
black
slack
shack
shark
share
shale
whale
while
white

p. 307 x=4

p. 311 1/6 km

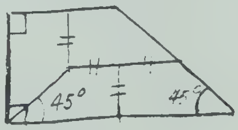
p. 315 10^{100} is one accepted meaning.

p. 318
2x4-6=2
3+7÷5=2
9-8+1=2

p. 324
50,100 ⟶
75
⟵ 50
50,75 ⟶

p. 330
5-6+7=6
9-2-1=6
8÷4x3=6

p. 334



p. 344
wheat
cheat
cleat
bleat
bleak
break
bread

p. 347
8-4+3-5
9+1-5=5
6x2-7=5

p. 351 W,G,L
$\xrightarrow{G}$
$\longleftarrow$ G
L $\xrightarrow{W}$
$\xleftarrow{G}$ W
G $\xrightarrow{L}$
$\longleftarrow$ W,L
$\xrightarrow{G}$

p. 355

p. 369
mine
mint
ment
meat
moat
coat
coal

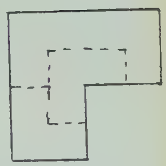
p. 370 $\frac{5\pi}{2}$ units

p. 375
6-7+5=4
1+9÷3=4
8x2÷4=4

p. 378
flour
foul
foil
fail
fall
fill
file
five

p. 383
7+8÷5
6-3÷1
9-4-2

p. 391



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